Kursawe's MOP is included because this two-objective function's P_{true} has several disconnected and unsymmetric areas in solution space. Its PF_{true} consists of three disconnected Pareto curves. Like MOP3, its solution mapping into dominated objective space is also quite convoluted. Like MOP2, its number of decision variables is arbitrary. However, changing the number of decision variables appears to slightly change PF_{true} 's shape and does change its location in objective space. We use it here with three decision variables. Note that both P_{true} (see Figure 4.7) and PF_{true} (see Figure 4.8) are disconnected. This function is renamed **MOP4**.

Viennet's MOP is proposed as the fifth generic test function since this tri-objective function's P_{true} consists of disconnected areas in solution space (see Figure 4.9). And its PF_{true} is a single, convoluted three-dimensional Pareto curve (see Figure 4.10). This function is designated as **MOP5**.

A MOP constructed using Deb's methodology (see Section 4.3.3) is selected. Like MOP4, this two-objective function's P_{true} and PF_{true} are disconnected, although its PF_{true} consists of four Pareto curves (see Figure 4.12). Its solution mapping into dominated objective space is not as convoluted as MOP4's. This problem is used to compare MOEA performance in finding similar phenotypes produced by different MOPs (c.f., MOP4). And this function is now called **MOP6**.

Finally, Viennet's second MOP is also suggested. This tri-objective MOP's P_{true} is a connected region in solution space (see Figure 4.11). Its PF_{true} appears to be a surface and its mapping into objective space appears straightforward (see Figure 4.12). This function is primarily meant to complement MOP5. This function is relabeled as **MOP7**.

Table 4.3: MOEA Test Suite Functions

MOP	Definition	Constraints
MOP1 P_{true} connected, PF_{true} convex	$F = (f_1(x), f_2(x))$, where $$f_1(x) = x^2,$$ $$f_2(x) = (x-2)^2$$	$-10^5 \leq x \leq 10^5$
MOP2 P_{true} connected, PF_{true} concave, number of decision variables scalable	$F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$f_1(\mathbf{x}) = 1 - \exp(-\sum_{i=1}^{n}(x_i - \frac{1}{\sqrt{n}})^2),$$ $$f_2(\mathbf{x}) = 1 - \exp(-\sum_{i+1}^{n}(x_i + \frac{1}{\sqrt{n}})^2)$$	$-4 \leq x_i \leq 4;\ i = 1, 2, 3$

Table 4.3: (continued)

MOP	Definition	Constraints
MOP3 P_{true} disconnected, PF_{true} disconnected (2 Pareto curves)	Maximize $F = (f_1(x,y), f_2(x,y))$, where $$f_1(x,y) = -[1 + (A_1 - B_1)^2 + (A_2 - B_2)^2],$$ $$f_2(x,y) = -[(x+3)^2 + (y+1)^2]$$	$-3.1416 \leq x,y \leq 3.1416$, $$A_1 = 0.5\sin 1 - 2\cos 1 + \sin 2 - 1.5\cos 2,$$ $$A_2 = 1.5\sin 1 - \cos 1 + 2\sin 2 - 0.5\cos 2,$$ $$B_1 = 0.5\sin x - 2\cos x + \sin y - 1.5\cos y,$$ $$B_2 = 1.5\sin x - \cos x + 2\sin y - 0.5\cos y$$
MOP4 P_{true} disconnected, PF_{true} disconnected (3 Pareto curves), number of decision variables scalable	$F = (f_1(\mathbf{x}), f_2(\mathbf{x}))$, where $$f_1(\mathbf{x}) = \sum_{i=1}^{n-1}(-10e^{(-0.2)*\sqrt{x_i^2+x_{i+1}^2}}),$$ $$f_2(\mathbf{x}) = \sum_{i=1}^{n}(\lvert x_i\rvert^a + 5\sin(x_i)^b)$$	$-5 \leq x_i \leq 5;\ i = 1,2,3$ $$a = 0.8,$$ $$b = 3$$
MOP5 P_{true} disconnected and unsymmetric, PF_{true} connected (a 3-D Pareto curve)	$F = (f_1(x,y), f_2(x,y), f_3(x,y))$, where $$f_1(x,y) = 0.5*(x^2+y^2) + \sin(x^2+y^2),$$ $$f_2(x,y) = \frac{(3x-2y+4)^2}{8} + \frac{(x-y+1)^2}{27} + 15,$$ $$f_3(x,y) = \frac{1}{(x^2+y^2+1)} - 1.1e^{(-x^2-y^2)}$$	$-30 \leq x,y \leq 30$
MOP6 P_{true} disconnected, PF_{true} disconnected (4 Pareto curves), number of Pareto curves scalable	$F = (f_1(x,y), f_2(x,y))$, where $$f_1(x,y) = x,$$ $$f_2(x,y) = (1+10y)* [1 - (\frac{x}{1+10y})^\alpha - \frac{x}{1+10y}\sin(2\pi qx)]$$	$0 \leq x,y \leq 1$, $$q = 4,$$ $$\alpha = 2$$

Table 4.3: (continued)

MOP	**Definition**	**Constraints**
MOP7 P_{true} connected, PF_{true} disconn ected	$F = (f_1(x,y), f_2(x,y), f_3(x,y))$, where $$f_1(x,y) = \frac{(x-2)^2}{2} + \frac{(y+1)^2}{13} + 3,$$ $$f_2(x,y) = \frac{(x+y-3)^2}{36} + \frac{(-x+y+2)^2}{8} - 17,$$ $$f_3(x,y) = \frac{(x+2y-1)^2}{175} + \frac{(2y-x)^2}{17} - 13$$	$-400 \leq x, y \leq 400$

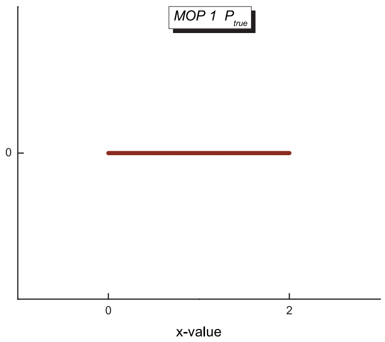

Fig. 4.1. MOP1 P_{true}

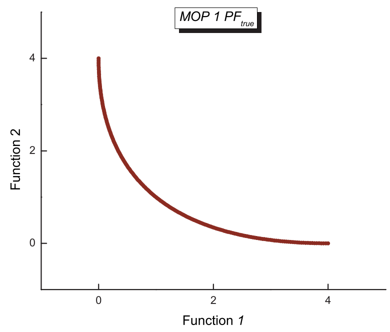

Fig. 4.2. MOP1 PF_{true}

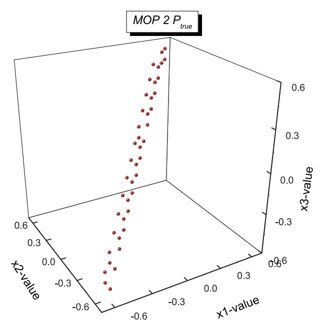

Fig. 4.3. MOP2 P_{true}

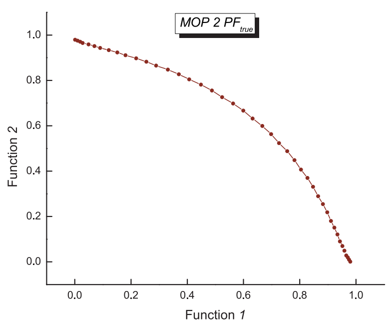

Fig. 4.4. MOP2 PF_{true}

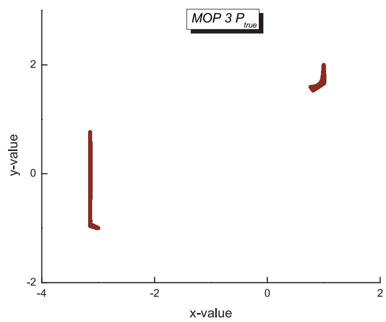

Fig. 4.5. MOP3 P_{true}

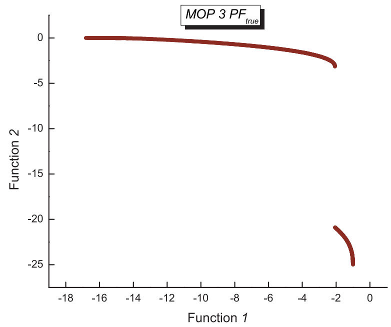

Fig. 4.6. MOP3 PF_{true}

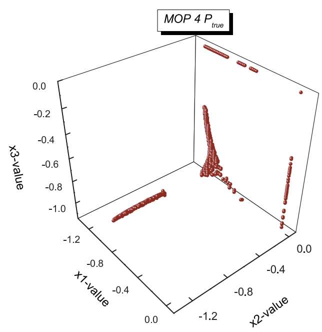

Fig. 4.7. MOP4 P_{true}

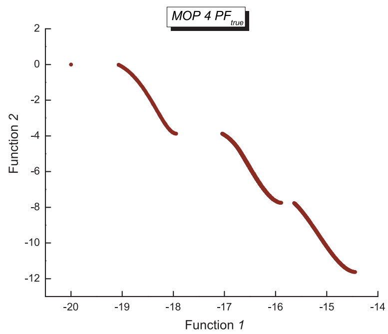

Fig. 4.8. MOP4 PF_{true}

These proposed numeric MOEA test functions in Table 4.3 address the issues mentioned in Section 4.2. MOP1 and MOP2 are arguably “easy” MOPs. MOP2 and MOP4 are scalable as regards decision variable dimensionality. MOP6 is scalable as regarding the number of Pareto curves in PF_{true} . MOP5 and MOP7 are tri-objective MOPs. All are nonlinear, and several show a lack of symmetry in both P_{true} and PF_{true} . Taken together these MOPs begin to form a coherent basis for MOEA comparisons. However, other relevant MOP characteristics (as reflected in Tables 4.1 and 4.2) may also be addressed by other MOPs selected for test suite inclusion. These additional MOPs may need to be constructed in order to exhibit some desired characteristics (see Section 4.3.3). Utilization of a test suite is advantageous to the community in the fact that it presents data that is baselined from a standard test suite [1633]. Another philosophical development of a test suite reflects similar functionality [1772].

Observe that parameters can be added to each function in order to highlight and emphasize PF_{true} characteristics providing more difficulty for a