Identification of a fitness function via a dynamical system generated by phenotypic evolution – a case study

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Abstract. Studies of a dynamical system model generated by a phenotypic evolution may be exploited to identify an unknown fitness function of a “black-box” type. Depending on a fitness function itself and a standard deviation of mutation, the system converges either to stable fixed points or demonstrates a periodic and/or chaotic behavior. Stable fixed points locate fitness optima while the unstable behavior may indicate asymmetry of the function. A family of bimodal tent functions are analyzed with their parameters varied, in order to gain knowledge about their optima positions and heights, saddles widths and levels.

1 Introduction

A behavior of a dynamical system of phenotypic evolution, obtained for very small populations and a real-valued infinite search space, seems to be a reliable tool of an unknown fitness function identification [4, 6]. Two-element populations evolving with proportional selection and Gaussian mutation are regarded in a space of population states. Expected values of the population states generate a discrete nonlinear two-dimensional dynamical system. An asymptotic behavior of the system depends both on a shape of a fitness function and on a parameter of evolutionary process - a standard deviation of mutation σ [2, 3, 5]. For small values of the standard deviation, the system converges to stable fixed points. When the parameter increases, some of the fixed points disappear, while some others change their stability. Bifurcations, periodic orbits and chaos may also be observed.

The analysis of stable and/or unstable behavior of the dynamical system may shed light on an unknown fitness function of “black-box” type or to design parameters of an evolutionary algorithm used in optimization [4, 6, 7]. A task of identification of an unknown function often occurs in engineering practice, where values of the function are obtained via mesurements. In the paper the extension of our past activities in the field of fitness functions identification is presented. An asymptotic behavior of the dynamical system is analyzed in details for bimodal fitness functions composed of two unimodal tent functions. The impact of position and height of optima for the fitness functions, as well as a width and level of a flat saddle, on fixed points and their stability of the system is studied. In a landscape of a bimodal function with optima separated by a flat saddle, the system behaves like in the case of an unimodal function: with increase of the standard deviation of mutation, stable fixed points are changed by an orbit of period of two. If a flat saddle level is high, this phenomenon is not observed. The chaos is detected for bimodal functions without flat saddles. Neither height of optima nor shift in their locations modifies qualitatively a behavior of the dynamical system.

The paper is organized as follows. In Section 2 the model of phenotypic evolution in the space of population states and the underlying discrete dynamical systems are recalled. Section 3 describes the framework and fitness functions exploited in simulations. The asymptotic behavior of the system for bimodal tent function is presented in Section 4. Section 5 provides a summary of obtained results.

2 Dynamical system model of two-individual population

An evolutionary search with soft selection is a phenotypic evolutionary method operating in a real-valued unbounded search space R^n [1]. Reproduction occurs with a proportional selection and the Gaussian mutation. Two-element population model of the method is considered [5]. Evolution of two-element population $P = \{\boldsymbol{x}_1, \boldsymbol{x}_2\}$ ($\boldsymbol{x}_i \in R^n$, $i = 1, 2$) is regarded in a *a space of populations state* S, where every point corresponds to the whole population. Assuming that the search space is one-dimensional, a population state can be represented in $R^2 \ni \boldsymbol{s} = \{x_1, x_2\}$. Because the ordering of individuals within a population is insignificant, an equivalence relation U is defined that glues points with permuted coordinates. Thus, the population space S is replaced by a factor space $S_U = R^2/U$ that is identified with the half-plane (bounded by line $x_1 = x_2$) $S_U = \{(x_1, x_2) : x_1 \geq x_2\}$. It is more convenient to rotate coordinates using the transformation $w = (x_1 - x_2)/\sqrt{2}$, $z = (x_1 + x_2)/\sqrt{2}$. Then, the state space S_U becomes the right half-plane ($w \geq 0$) bounded by Z-axis and the population in i-th generation is described by a state $\boldsymbol{s}^i = (w^i, z^i)$. Coordinate w corresponds to population diversity while z – to the population mean.

In S_U, expected values of the population state can be calculated as it was done in [5]. The final expressions of expected values in the $(i+1)$-st generation for w and z are given

$$\begin{cases} E_{i+1}[w|\boldsymbol{s}^i] = & \sqrt{\frac{2}{\pi}}\sigma + (1 - {\Psi^i}^2) \cdot \sigma \cdot \Theta(w^i/\sigma) \\ E_{i+1}[z|\boldsymbol{s}^i] = & z^i + \Psi^i \cdot w^i, \end{cases} \tag{1}$$

where $q(x)$ denotes a fitness of the individual x and

$$q_1 = q(x_1) = q\left(\frac{w+z}{\sqrt{2}}\right), \; q_2 = q(x_2) = q\left(\frac{z-w}{\sqrt{2}}\right), \; \Psi(w,z) = \frac{q_1 - q_2}{q_1 + q_2}, \; \Psi^i = \Psi(w^i, z^i),$$
$$\Theta(\xi) = \phi_0(\xi) + \xi\Phi_0(\xi), \;\; \phi_0(\xi) = \frac{1}{\sqrt{2\pi}}(\exp\left(-\frac{\xi^2}{2}\right) - 1), \;\; \Phi_0(\xi) = \frac{1}{\sqrt{2\pi}} \int_0^\xi \exp(-\frac{t^2}{2})dt.$$

Expected values (1) generate a two dimensional discrete dynamical system

$$(w, z) \longrightarrow F(w, z) = \begin{bmatrix} F_1(w,z) = E_{i+1}[w|\boldsymbol{s}^i] \\ F_2(w,z) = E_{i+1}[z|\boldsymbol{s}^i] \end{bmatrix}. \tag{2}$$

The asymptotic behavior of dynamical system (2) was studied and fixed points and their stability were determined. Fixed points (w^s, z^s) of the system are characterized by

$$w^s \simeq 0.97\sigma, \quad \Psi(w^s, z^s) = 0.$$

Recent studies demonstrate that, in general, fixed points of the system (2) are located in vicinity of optima and saddles of a fitness function. Saddle fixed points are unstable. The optima points are stable for small values of the standard deviation of mutation and may loose their stability for larger σ. Then periodical orbits and/or chaos appeared. Till now, symmetrical and asymmetrical, unimodal and bimodal Gaussian functions and unimodal tent functions were studied [2, 4, 6]. Unimodal fitness functions have at most one fixed point localizing the optimum. The fixed point is stable for small values of the standard deviation of mutation. For large σ the stable fixed point disappears and a periodical orbits for symmetrical functions or period doubling bifurcations leading to chaos for asymmetrical ones arise. For bimodal Gaussian fitness functions, the dynamical system has up to three fixed points. Points are located near optima and a saddle. Optima fixed points are stable for small σ whereas a saddle fixed point is unstable. When the value of standard deviation of mutation is increased, the system looses stability and an orbit of period two is observed for symmetric functions or asymmetric functions with hills of equal width and different optima height. For functions with different hills width unstable chaotic behavior was detected.

Keeping trace over the asymptotic behavior of the dynamical system may be useful in setting parameters of the soft-selection based evolutionary algorithms when applied to optimization problems. The analysis of the behavior may prove its usefulness in an identification of an unknown fitness function. The number of fixed points is tightly correlated with modality of the function. For small σ, fixed points reliably localize optima while chaotic behavior (for larger σ) may indicate the asymmetry of a fitness function.

3 Prerequisites

In simulations expected values of coordinate z were calculated until either its stable state was reached or an assumed number of iterations were performed. The stable state was called out when the values of z in two consecutive iterations differed less than the prescribed tolerance, $\epsilon = 10^{-6}$. The maximal number of iterations was fixed on 5000. The stable value of coordinate w: $w_s \cong 0.97\sigma$, in computations was rounded to $w_s = \sigma$. Values of the standard deviation of mutation were uniformly taken from range $[0.05, 2]$ with the step $\Delta\sigma = 0.005$. Initial states $s_0 = (w_0, z_0)$ and $s_0 = (w_0, -z_0)$ were generated setting $w_0 \in \{0.1 + i \cdot 0.2\}, i = 0, 1, \ldots, 4$, and $z_0 \in \{0.1 + i \cdot 0.2\}, i = 0, 1, \ldots, 8$.

A bimodal fitness function is composed of two tent functions separated by a flat saddle. It admits to set easily various values of optima heights and localizations, a saddle width and level (Fig. 1.a.)

$$q = \begin{cases} ((h_1 - h_v)/(B - A)) \cdot (x - A) + h_v & \text{if } x \in [A, B) \\ ((h_s - h_1)/(C - B)) \cdot (x - B) + h_1 & \text{if } x \in [B, C) \\ h_s & \text{if } x \in [C, D) \\ ((h_2 - h_s)/(E - D)) \cdot (x - D) + h_s & \text{if } x \in [D, E) \\ ((h_v - h_2)/(F - E)) \cdot (x - E) + h_2 & \text{if } x \in [E, F] \\ h_v & \text{otherwise} \end{cases} \tag{3}$$

where $A < B < C \leq D < E < F$, $h_1 > h_s$, $h_2 > h_s$. For convenience $l_s = D - C$ stands for the saddle width. If not mentioned opposite, default values of parameters are the following: $h_1 = 2$, $h_2 = 1$, $h_s = 0.1$, $h_v = 0.001$, $l_s = 2$. Various shapes of the fitness were considered: functions with or without flat saddle (Fig. 1.b), functions with flat saddle of different level (Fig. 1.c), functions with optima of equal or different height (Fig. 1.d) and function (Fig. 1.a) shifted along the x axis.

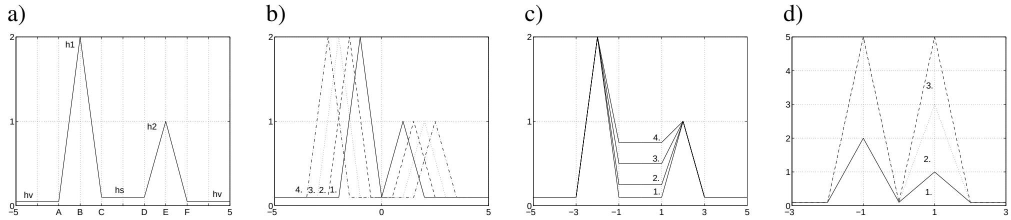

Figure 1. Bimodal fitness functions: a) a general form; b) functions with different saddle width ($h_v = h_s = 0.1$, $h_1 = 2$, $h_2 = 1$), 1. $l_s = 0$, 2. $l_s = 1$, 3. $l_s = 2$, 4. $l_s = 3$; c) functions with different saddle level ($h_v = 0.1$, $l_s = 2$, $h_1 = 2$, $h_2 = 1$), 1. $h_s = 0.1$, 2. $h_s = 0.25$, 3. $h_s = 0.5$, 4. $h_s = 0.75$; d) functions with optima of equal or different height ($h_s = h_v = 0.1$, $l_s = 0$), 1. $h_1 = 2$, $h_2 = 1$, 2. $h_1 = 5$, $h_2 = 3$, 3. $h_1 = 5$, $h_2 = 5$

4 Behavior of the dynamical system - results

In general, behavior of the system is independent of an initial value of coordinate w, which describes a diversity of population. It is understandable when looking at evolution of expected value of coordinate w [4, 6]. Its value is bounded, from above and below, and very quickly attains its stable value. The value of $E[z]$ corresponds to an expected location of population and thus influences behavior of the system. In Fig. 2.a bifurcation diagrams for the function without a flat saddle ($l_s = 0$) are presented. A single fixed point, for small values of the standard deviation of mutation, is replaced by periodical orbits and chaos when the parameter increases. Stable fixed points locate both fitness optima. For all three tested values of coordinate w_0, diagrams look similar to each other and differ only in values of σ for which the unstable behavior appeared. Detailed diagrams for $w_0 = 0.5$ are presented in Fig. 2.b. Depending on an initial value of z_0, different optima are located i.e. initial state $\boldsymbol{s}_0 = (0.5, 0.1)$ is situated in the attraction basin of the global optimum whereas trajectories started at the other states are attracted by the local optimum and changed attractor for larger σ. Later on bifurcation diagrams for positive values of z_0 are mainly presented because for negative z_0 they are symmetrical. Exemplary diagrams for both positive and negative z_0 are displayed in Fig. 4.

Flat saddles width. The effect of a flat saddle between optima is analyzed for functions with various saddle widths ($l_s = 1, 2, 3$) and compared with the fitness without a flat saddle (Fig. 3). For a fitness with a flat saddle both initial coordinates w_0 and z_0 may effect expected trajectories depending on localization of individuals corresponding to the initial state. Trajectories started in a state for which both of individuals (x_1 and x_2) are placed on a flat saddle remain in the state because there is no difference in fitness of the individuals, $\Psi = 0$ and from (1): $E_{i+1}[z] = z^i$. Stable fixed points located optima unless one of individuals is situated on an optimum hill. For example: $\boldsymbol{s}_0 = (0.5, 1.1)$ corresponds to population $P = (1.13, 0.42)$ for which individual x_1 is situated on the local hill of the function (3) with $l_s = 2$. Trajectories started at this state converge to fixed points near the local optimum. For saddles of width $l_s = 1, 2$ and 3 stable optima fixed points loose stability for large σ and pitchfork bifurcations give a rise to orbits of period of two (Fig. 3). The doubled period bifurcations and chaos are detected only for fitness without a flat saddle (Fig. 2). This effect may be explained easily. For functions with a flat saddle optima are separated. They do not influence each other as in the

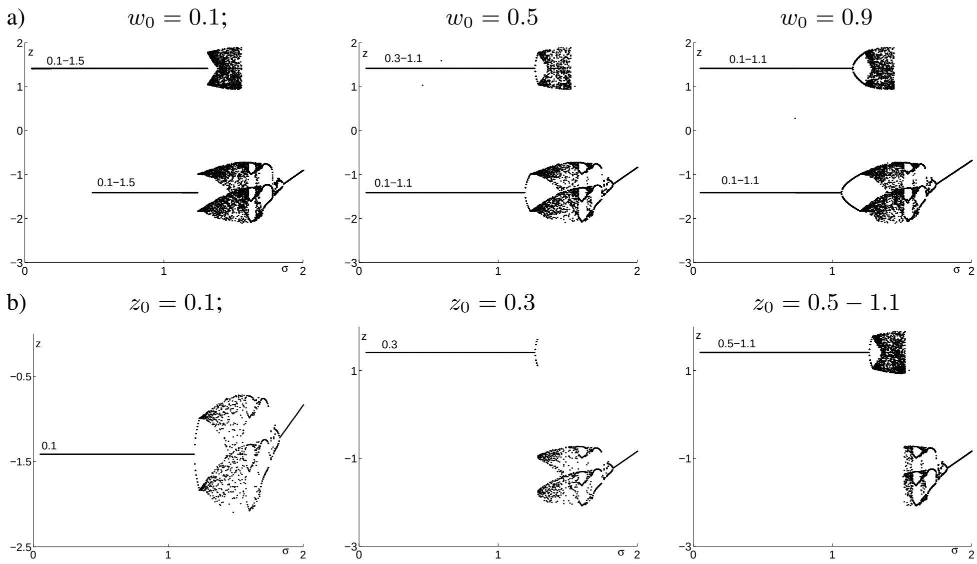

Figure 2. Bifurcation diagrams for fitness (3) with no flat saddle ($l_s = 0$) and various initial states $\boldsymbol{s}_0 = (w_0, z_0)$: a) initial states with $w_0 = 0.1, 0.5, 0.9$ and various values of z_0 given in diagrams; b) initial states with fixed $w_0 = 0.5$ and various values of z_0; $h_1 = 2$, $h_2 = 1$

case of function without flat saddle and may be regarded as a single unimodal symmetric hills. For small σ the other hill is not "seen" and the dynamical system behaves as in the unimodal case.

Flat saddles level. The effect of a saddle level is analyzed for a flat saddle with various level $h_s = 0.1, 0.25, 0.5, 0.75$ (Fig. 4). Because diagrams for different values of w_0 look similar, see Fig. 2, 3, remaining results are presented only for $w_0 = 0.5$. For $h_s = 0.25$ and $h_s = 0.5$ figures for both positive and negative values of z_0 are shown. Elevation of a saddle causes that bifurcations arising for large σ and initial individuals located on local or global optimum hill are less distinct and for higher saddle levels utterly disappeared. In that case trajectories converge to fixed points located on hills' slopes separated by a saddle (which are not as steep as the others). A lack of bifurcations for higher saddle levels may be explained by smaller difference in fitness of individuals located on hills and on saddle, thus trajectories can stabilized on slopes.

Optima height. The fitness function with and without flat saddles and various optima heights: different heights $h_1 = 2$, $h_2 = 1$ and $h_1 = 5$, $h_2 = 3$ and equi-height hills $h_1 = h_2 = 5$, were regarded (Fig. 1.d). Results presented in Fig. 5 show that influence of optima height is negligible. For symmetrical function without a flat saddle ($l_s = 0$, $h_1 = h_2 = 5$) and a given initial state only one optimum was detected.

Shift along x axis. Functions with different localization of optima (positions of the global optimum $B = -1, 0, 1$) were regarded. General pattern of diagrams is analogous. Because initial states for all functions were the same, only the global optimum was detected for the function with $B = 1$ (location of the local optimum required larger values of z_0). Interesting

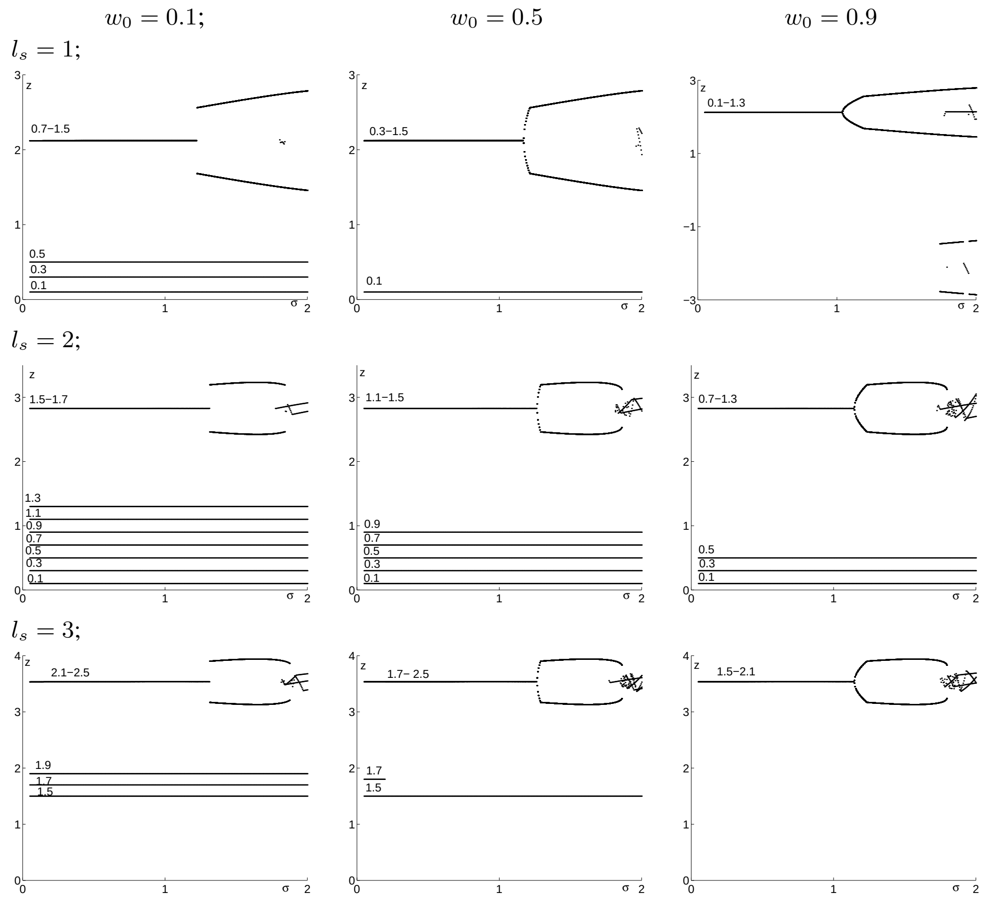

Figure 3. Bifurcation diagrams for fitness (3) with different saddle width l_s and various initial states $\boldsymbol{s}_0 = (w_0, z_0)$; values of z_0 are given in diagrams; $h_1 = 2$, $h_2 = 1$, $h_s = 0.1$

and worth of further studies is an example of the function with the global optimum located at zero: chaos lasts for small range of σ and then the system converges to one stable fixed point again and switches to orbit of period of 2 later on.

5 Conclusions

A case study of the dynamical system generated by phenotypic evolution for bimodal fitness functions composed of two tent functions was presented. Different parameters constituting the fitness function were examined in context of using the results in identification of an unknown fitness function.

It turned out that studies of fixed points can be carry out for one value of coordinate w_0 only, varying values of z_0 in a wide range. Generally, for small values of standard deviation of mutation σ, fixed points locate optima of a fitness. Fixed points lazy stayed at an initial

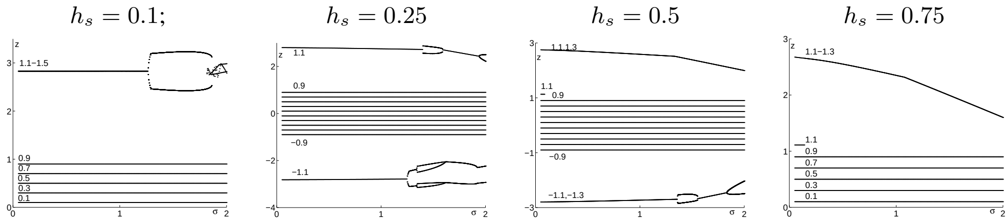

Figure 4. Bifurcation diagrams for fitness (3) with different saddle levels and various initial states $\boldsymbol{s}_0 = (0.5, z_0)$; values of z_0 are given in diagrams; $h_1 = 2$, $h_2 = 1$, $l_s = 2$

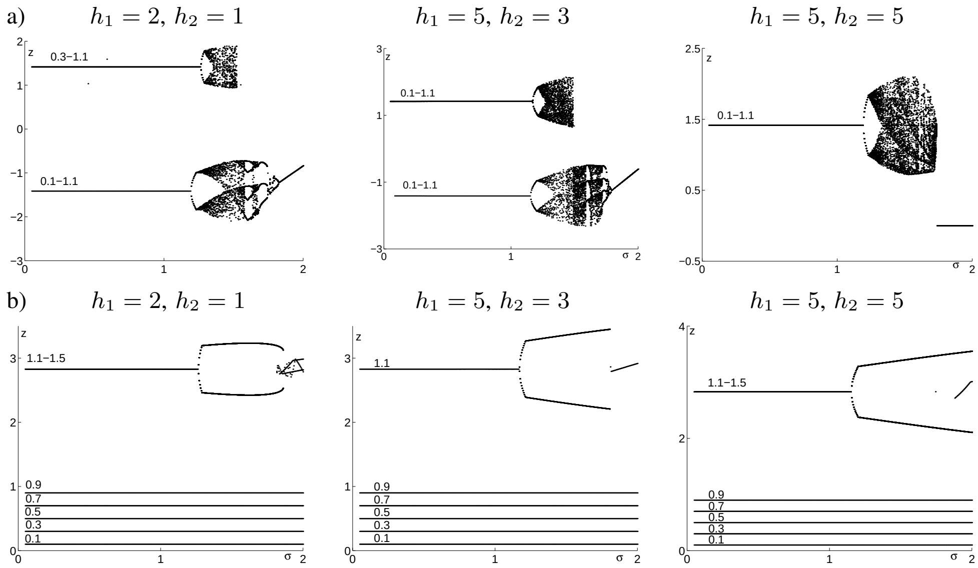

Figure 5. Bifurcation diagrams for fitness (3) with different optima height and various initial states $\boldsymbol{s}_0 = (0.5, z_0)$, values of z_0 are given in diagrams; a) $l_s = 0$; b) $l_s = 2$

state may indicate a flat surface of a function (saddle or plateau). In this case, fixed points may change position and locate an optimum if trajectories started at states for which at least one individual is at the optimum peak. Thus, such an initial state can serve as a hill indicator. Small differences in fitness between individuals on hills and on a flat saddle (functions with a high saddle level) may cause that unstable behaviors (orbits and/or chaos) do not appear. Chaos was detected for functions without a flat saddle where there is a mutual influence of both optima on the system. Different optima height and the shift of the function along x axis have a minor effect on the system behavior. The presented case study seem to confirm our earlier observations, gained from the analysis of Gaussian-like uni- and bi-modal fitness, that following the asymptotic behavior of dynamical system (2) is a useful tool in identification of “black-box” fitness functions.

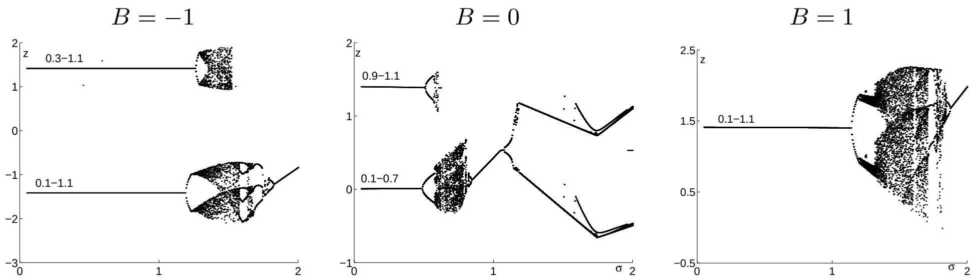

Figure 6. Bifurcation diagrams for fitness (3) shifted along x axis and various initial states $\boldsymbol{s}_0 = (0.5, z_0)$, values of z_0 are given in diagrams; $l_s = 0$, $h_1 = 2$, $h_2 = 1$

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