Evolutionary algorithm for market simulations

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Abstract. Evolutionary algorithms (EA) have recently become not only tools for efficient optimization of very difficult problems, but also are applied to simulate behavior of different kinds of systems, among them also games, economic systems and markets. This new domain of EA applications is known as Agent-Based Computational Economics (ACE). This article describes two applications of EA to simple market simulations. The main aim of EA in this approach is to find (sub-) optimal strategies of behavior for the participants of that market game. The first example is a simple market with only several participants and one product, well known as an instance of Cournot oligopoly game. The second example is more complicated and describes a market of permits for CO_2 emission, created by the Kyoto Protocol and introduces to the simple Walrasian model the influence of calculated on-line permits prices.

1 Introduction

Application of EA for economic models simulation (ACE [6]) focused a big attention of researchers in that domain, mainly due to the fact that economic systems may be quite easily modeled using EA. Members of EA population are treated as agents, which learn to behave almost optimally and adopt their strategy to get higher income, playing its "market-game". Also evolutionary operators have new interpretation in this approach. Mutation-like, exploring operators are instances of learning by experiments while exploitation operators similar to crossover are treated as learning by imitation [6, 13].

Generally, there are two approaches to economic modeling using EA. The first one treats economic agents taking part in the market directly as members of the EA population [1]. The second one assumes that one member of the EA population is an instance of the whole market with all its participants [1] and its chromosome is a game strategy. There are different advantages and disadvantages of both approaches. The most important shortcoming of the first one is potentially too small or rarely too big number of population members, depending on the problem size, but it is easy to simulate market participants with their private preferences and quality functions. Thus, it was used to simulate the Cournot oligopoly market. The second method is more independent on the solved problem, but it rather requires a global quality function for the whole optimized market and thus it was used to solve the CO_2 permits trading problem.

The Cournot oligopoly game is a well-known example of a market game in which participants compete on the amount of total production of some commodity, trying to maximize their benefits, making decisions on the production quantity. This problem is the first testing example considered in simulations.

The second task is the model of CO_2 permits market, which became as a consequence of signed in 1997 the Kyoto Protocol. To know the influence of Kyoto Protocol limitations on world economy, researchers from different countries want to build a model of such market and find optimal selling/buying strategies for their countries. The market model, which enables to forecast quantities and prices of traded emissions allowances and the cost of emission reduction for different countries is very necessary. Important problem is to build a transaction model and to solve many other problems associated with emission level reports credibility and uncertainty [4, 8, 10, 12]. Proposed in this paper a new idea of problem solving is different that the ideal market situation. Therefore more sophisticated market model was introduced, where some typical elements of real market were added: the possibility of price negotiation and the influence of real prices on obtained financial results. Next parts of the paper present EA used to simulate both markets and results of the simulations with conclusions at the end.

2 Modeled markets

2.1 The Cournot oligopoly game

The Cournot oligopoly game is an example of a small market where only a few firms (fixed number, in conducted tests four firms) compete on one homogeneous product, making decisions of the quantity of their production (q_i), considering information from other firms. Firms are not allowed to cooperate. Every firm wants to maximize its profits (π_i) and tries to find the best strategy – quantities of its production. There are only a few firms in the market to assess their influence on it (market power). The price of the commodity depends on total production (1) and profit depends on price and production (2):

$$P(t) = M - a \cdot Q(t) = M - a \cdot \sum_{i=1}^{n} q_i(t) \tag{1}$$

$$\pi_i(t) = P(t) \cdot q_i(t) - b \cdot q_i(t) \tag{2}$$

P(t) – price of produced commodity; *Q(t)* – total quantity of production; $q_i(t)$ – production of firm *i*; $\pi_i(t)$ – profit of firm *i*; *t* – time factor; *n* – number of market participants; *M*, *a*, *b* – some constant market parameters.

This small example was applied as a test-base for more sophisticated problem concerning trading of CO_2 permits, which is described in the next section.

2.2 The model of CO_2 permissions for emission market

The Kyoto Protocol imposing constraints on CO_2 emissions of participants, gives also some opportunities to exceed them. There are usually countries that can't exhaust their limitations – they can sell them to different countries and due to this fact a market of emission permits trading becomes. Trading is beneficial only when the price of permits is lower than the cost of emission reduction for the same amount of CO_2. Thus, the country which wants to offer some permits on the market can decrease its emission level even more than its obligation and sell remaining permits (see Fig. 1), but of course it must be a beneficial transaction and selling permits should bring more money than spending them on emission abatement.

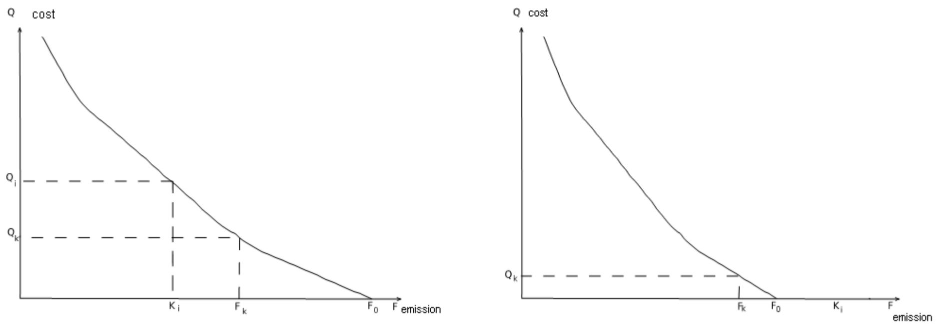

Figure 1. Emission reduction cost for buying country (left): without trade (Q_i) and with trade (Q_k) for buying country, K_i – Kyoto limit, F_k – emission after trade, F_0 – initial emission and emission reduction cost for selling country (right): without trade is equal zero ($F_0<K_i$) and after trade is equal Q_k with emission F_k for selling country; K_i – Kyoto limit, F_0 – initial emission.

The simple and commonly used Walrasian model[1] of emission market denotes the total cost of decreasing emission in region *i* down to x_i, by $C_i(x_i)$, (the abatement cost function). It is usually assumed that cost functions $C_i(x_i)$ are positive, decreasing and continuously differentiable for each region. The Kyoto constraint imposed on region *i* is indicated by K_i. A number of emission permits acquired by source is expressed by s_i (s_i is negative if region *i* is a net supplier of permits). The goal is to minimize the reduction cost (3) to obtain the Kyoto target (4) fulfilling needs of participants without any extra permits (5).

$$E = \min_{x_i} \sum_{i=1}^{n} C_i(x_i) \tag{3}$$

$$x_i \leq K_i + s_i \tag{4}$$

$$\sum_{i=1}^{n} s_i = 0 \tag{5}$$

E – quality function of the base model, $C_i(x_i)$ – the costs of decreasing emission at region (country, source) *i* from initial value F_{0i} down to x_i, s_i – the *number* of permits acquired by region *i*, K_i – Kyoto target for region *i*, *n* – number of regions, x_i – current emission.

Normally prices (shadow price) are defined as the cost derivatives in a given point, but typically the cost reduction function and exact reduction cost are not known or are known very imprecisely [12]. Even if they were known, they could not be directly applied, because they are not the only one component of emission permits' price. More factors are typically influencing prices of goods and the same mechanisms may have role in the permits market. In the described approach the most important factors are of course estimated emission reduction costs. The upper limit of price is a buyer emission reduction cost, the lower limit is a seller emission reduction cost. Thus, the transaction price must be between them (in the opposite

[1] Walrasian trade model claims that prices are calculated by some market authority on the basis of supply and demand in a market and all transactions are conducted according to this price.

case transaction is not profitable for one participant). It is assumed that transaction is finalized only when permit price, which was negotiated, is lower than the cost of reduction for the buyer, and higher than the cost of the seller. It is obvious that each party wants to maximize its profit. In the further described evolutionary approach, maximization over s and π is performed for each transaction by genetic operators, while the total maximization over x is the EA task. Described assumptions become a basis for a new model with quality function (6).

$$G = \max_{x} \sum_{j=1}^{T} \sum_{i=1}^{n} \max_{s_{ji}\pi_{ji}} \left(C_{j-1,i}(x_{j-1,i}) - \left(C_{ji}(x_{ji}) - s_{ji} \cdot \pi_{ji} \right) \right) \tag{6}$$

$$\sum_{j=1}^{T} x_{ji} \leq K_i + \sum_{j=1}^{T} s_{ji} \tag{7}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{T} s_{ji} = 0 \tag{8}$$

G – quality function of the new model, T – number of conducted transactions of permits buying/selling, $C_{ji}(x_{ji})$ – the costs of decreasing emissions at region i from initial value F_{0i} to value x_{ji} after j transactions, K_i – Kyoto target for region i, n – number of regions, x_{ji} – current emission, s_{ji} – the number of emission permits acquired by region i at transaction j, π_{ji} – price of permits bought/sold.

The new model described by formulae (6-8) maximizes the difference between cost with no trade and cost in case of trade plus expenditures for the permits. It allows to include also buying/selling permit price, which considerably influences transaction profitability and decision to buy/sell permits or to reduce emissions rather than to buy permits. Thanks to new function it is possible to find a solution, which maximizes the profit from emission trading. In the previous model's goal function (3) the cost of emission reduction without including any buying prices and expenditures for this goal is minimized (Walrasian model, which assumes that transactions are conducted with previously calculated optimal price, the same for all market members), but the cost of buying can be considerable, in comparison to expenditures for CO_2 reduction if there is no trade. Also a different method of permit price setting is accepted. The participants of the market must set the minimal price, below which price permit cannot decrease to avoid a case when country, which reports emission below the Kyoto level has zero marginal cost of abatement (comp. Fig. 1). Therefore marginal cost (e.g. shadow price) is not a derivative of abatement cost, but derivative with minimal value. The real price of permit and number of traded permits is not known, before computer simulation of market activity.

The second important change is introducing of transactions. Transactions are conducted iteratively until no one can be conducted (due to lack of benefit for both participants). Prices and amounts of transferred permits are negotiated. Thus, present market model is dynamic, contrary to static base one.

3 Evolutionary methods used to simulate described markets

3.1 Evolutionary simulation of the Cournot oligopoly game

Evolutionary simulation of the Cournot oligopoly game uses a specialized evolutionary method, based on solutions described in literature [1, 13], but with some new extensions. These extensions are: specialized genetic operators, encoding method and ranking of applied market strategies. First of all it must be noticed that simulation method is not a typical evolutionary algorithm, but some kind evolutionary method described in Algorithm 1.

1. Random initialization of strategies of the population of market players.
2. Modification of market players' strategies using specialized genetic operators.
3. Valuation of obtained results and modification of the players' sets of experiences.
4. Selection of new strategies on the basis of stored data.
5. If a stop condition not satisfied, go to 2.

Algorithm 1. The evolutionary method of market game simulation.

As it is possible to see in the Algorithm 1, the evolutionary method does not have a selection of individuals, except it the selection of strategies is introduced. In this case the notion "strategy" means simply a quantity of production in current game.

Each player conducts it own calculations during the repeated games and tries to optimize its profits or in different words, its own quality function. All players create one market, but are treated as separate population members, thus they can develop independently, optimizing their own strategies. In the approach with many evolving markets, players are only parts of one population member (market) and it would be difficult to value them separately, because one population member has usually one optimized criterion

To learn its strategy, the player stores data about production (strategies) and obtained profits (sums of obtained incomes) in past games and according to this experience it usually selects high valued strategies. The data structure describing one player contains:

- a value of current production (strategy);
- a vector of all used strategies (they are integer and limited to the set of values 0...M), but not all strategies are usually applied;
- a vector of real values of obtained results of the same length as the previous one, where the strategy from the first vector corresponds with sum of obtained profits from the second vector;
- a vector of integer numbers, which describes numbers of applications of all used strategies.

Genetic operators are selected to modify strategies using special adaptation method, which uses reinforcement learning and is described in [5, 11, 14]. Genetic operators mainly randomly modify the selected strategy and this becomes a player's bid for the next game. A set of genetic operators contains:

- random value generation (0...M);
- small random modification of selected strategy;
- a (slightly modified) copy of some previously used strategy of one of the players.

3.2 Evolutionary algorithm for simulation of the CO_2 emissions market

In this case a standard evolutionary algorithm is used, which works in the manner as it is shown in the Algorithm 2, with some adjustments to the solved problem.

1. Random initialization of the population of solutions.
2. Reproduction and modification of solutions using genetic operators.
3. Valuation of the obtained solutions.
4. Selection of individuals for the next generation.
5. If a stop condition not satisfied, go to 2.

Algorithm 2. The evolutionary algorithm.

One population member contains information about all the countries participating in the market, so it is a complete solution of the problem. It is possible to create as many individuals as needed (in the simulations about 400 are used) and obtain the same number of mainly different solutions. Of course the best one is the most important, but in some circumstances several of the remaining solutions can be used. Information needed to describe one country is encoded as a vector of 8 real-valued numbers:

- theoretical price of own permits (shadow price);
- the real price of current sold/bought permit;
- the value of current sold/bought permits;
- number of currently sold/bought permits;
- the total sum of sold/bought permits;
- current emission;
- previous emission (before present transaction);
- value of present and previous goal function

The population member contains the same number of such vectors as a number of market participants. To modify solution, the following genetic operators were used:

- competition – the chosen country set some numbers of permits for sale, and the other set offers to buy, when the best option is chosen, the solution is modified;
- bilateral sale – the randomly chosen two countries conduct transaction and if they make a deal, profitable for both of them, the solution is modified.

Prices and numbers of traded permits are randomly chosen. The number of traded permits is chosen from interval {1,..,5}, and the permit price is as a value between buying offer and sale offer with the expected value as average of these two values (simulation of the negotiation process). The fitness function for EA is directly a problem's quality function described by formula (6). The fitness function does not have any punishment part for constraints (7) and (8) violation, because forbidden solutions are not produced by initializing function or genetic operators.

4 Results of computer simulations

4.1 Results for the Cournot oligopoly game

Computational tests were conducted for parameters n=4, a=1, b=56, M=256 of equations (1) and (2). Four firms compete on the same product, trying to maximize their profits. Results of 10^5 games are presented in Fig. 2, for the best and the worst player, but remaining ones are quite similar (but not identical, as it can be seen in Tab. 1).

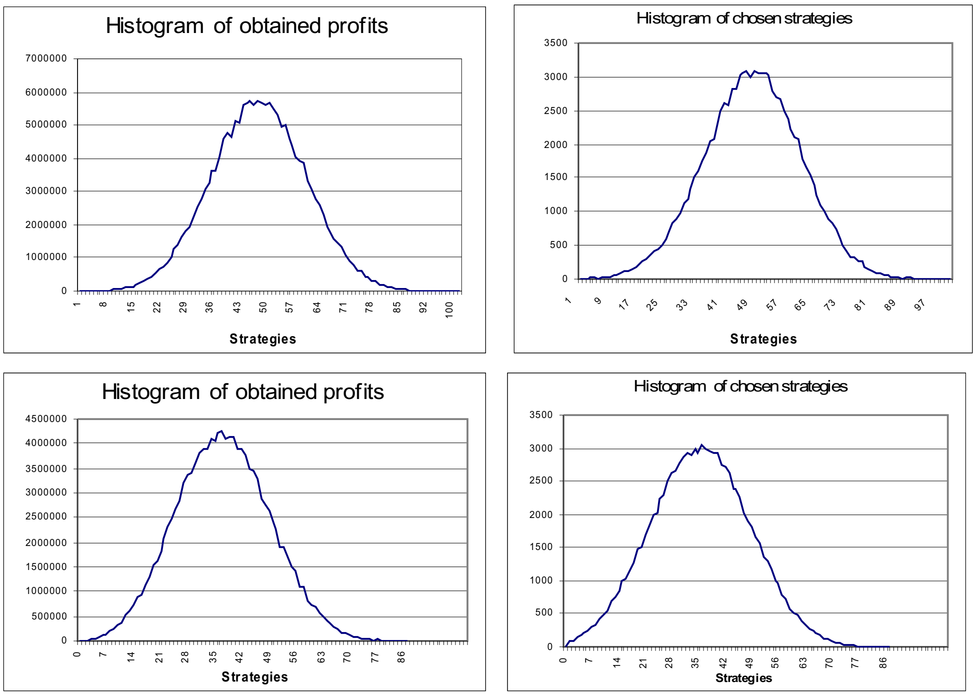

Figure 2. Histograms of obtained profits and chosen strategies obtained for player I (up) and II (bottom) after 10^5 iterations.

In the discussed case the problem has three equilibriums [1]:

- collusion equilibrium, where firms act as a single firm with q_i=25 and π_i=2500;
- Nash equilibrium with q_i=40 and π_i=1600;
- competitive equilibrium with q_i=50 and π_i=0.

As it can be seen in Fig. 2, the best player the most frequently selects strategies that are located between Nash and competitive equilibrium with average profits a little higher than for Nash equilibrium. The worst player frequently chooses strategies with a little smaller production than required for the Nash point and obtains significantly worse results than the best player.

Table 1. Average profits obtained by players after 10^5 games.

Player	**I**	**II**	**III**	**IV**
Average profit	1789.03	1299.29	1495.60	1317.14
The most freq. used strategy	46	36	40	36

4.2 Results for CO_2 emission permit trading problem

Computer simulations were conducted on a standard data set, similar to works [3, 9, 12]. The following countries: USA, EU, Japan, Canada-Australia-New Zealand (CANZ) and Former Soviet Union (FSU) are taken into account. The data presented in this work are rather approximate. For instance they consider data for USA, but this country hasn't signed the Kyoto protocol yet. Though, it would be difficult to practically start the CO_2 permissions market omitting the country with the biggest CO_2 emission level in the world. Thus, the USA are usually considered in simulations, but this causes the fact that real prices of permits are smaller than obtained from different models considering the USA presence, because of significantly smaller demands.

The emission abatement costs depend on value of emission reduction according to the formula (9) (quadratic cost function) [3, 9]. The marginal price (10) is a derivative of cost function, with a small modification of both component formulae (for $x<F_0$ and for $x\geq F_0$) – introduction of value *min_p* which is a minimal price of allowances, prevents the situation where permits are sold with price 0, which may occur when costs of emission reduction are 0 for a country with $F_{0i}<K_i$: Table 2 describes coefficients of the cost function (9) and shadow price (10) for considered countries.

$$C(x) = \begin{cases} a\cdot(F_0 - x)^2 & \text{for} \quad x < F_0 \\ 0 \quad \text{for} \quad x \geq F_0 \end{cases} \tag{9}$$

$$c(x) = \begin{cases} \max\left(\left(2\cdot a\cdot(F_0 - x)\right), min\_p\right) & \text{for} \quad x < F_0 \\ min\_p \quad \text{for} \quad x \geq F_0 \end{cases} \tag{10}$$

min_p – minimal price of permits, a – cost function parameter; F_0 – initial emission; x – actual emission.

Table 2. Data applied for calculations.

Country (region)	Initial emission (F_0) MtC/y	Cost function parameter (a) MUSD/(MtC/y)^2	Limit Kyoto (K_i) MtC/y
USA	1820.3	0.2755	1251
EU	1038.0	0.9065	860
Japan	350.0	2.4665	258
CANZ	312.7	1.1080	215
FSU	898.6	0.7845	1314

Table 3. Results after assuming perfect permit market model (the column "Final price" denotes the shadow price in the equilibrium point).

Country (region)	Final Emission MtC/y	Final price USD/tC	Number of imported permits Mt/y	Permits expenditures MUSD/y	Emission reduction cost MUSD/y
USA	1562	143	310	11974.3	18523.7
EU	959	143	100	15790.6	5515.1
Japan	321	143	63	29987.6	2074.3
CANZ	248	143	33	16077.6	4638.2
FSU	808	143	-506	-73830.3	6439.5

Table 4. The results of simulations from the new model (the column "Final price" denotes the shadow price in the point where trade finished due to lack of benefit) for the assumption that the seller imposes prices.

Country (region)	Final Emission MtC/y	Final price USD/tC	Number of imported permits Mt/y	Permits expenditures MUSD/y	Emission reduction cost MUSD/y
USA	1562.0	142.3	311	50222.9	18381.1
EU	959.0	143.2	99	15014.8	5657.5
Japan	321.0	143.1	63	14420.0	2074.3
CANZ	248.0	143.4	33	1741.7	4638.2
FSU	808.0	142.2	-506	-81399.3	6439.5

Table 5. The results of simulations from the new model (the column "Final price" denotes the shadow price in the point where trade finished due to lack of benefit) for the assumption that buyers impose prices

Country (region)	Final Emission MtC/y	Final price USD/tC	Number of imported permits Mt/y	Permits expenditures MUSD/y	Emission reduction cost MUSD/y
USA	1562.0	142.3	311	32055.6	18381.1
EU	959.0	143.2	99	10593.3	5657.5
Japan	321.0	143.1	63	6760.6	2074.3
CANZ	248.0	143.4	33	1695.1	4638.2
FSU	808.0	142.2	-506	-51104.6	6439.5

Application of EA method to simulate the permits market gives some additional benefits, because the result is not the only one set of parameters, but a set of possible scenarios. EA operates on a population of mostly different solutions and computations are conducted in non-deterministic way. Especially negotiation of permit's prices are modeled as random numbers with little modified normal distribution (prices are generated from the interval which is profitable for both countries, if there is no such interval, no transaction is made), thus different scenarios depend mainly on negotiated (i.e. randomly generated in the simulation) prices. This is also the reason of presentation of obtained results in two tables. Table 4 presents results from the scenario with high prices, imposed by sellers, Tab. 5 with low prices, imposed by buyers. As it is possible to notice, differences between them are not very high, except the columns "Permits expenditures". Results were selected from 10 conducted simulations.

Analyzing the data in Tab. 3 (old model), 4 (new model with high prices), and 5 (new model with low prices), it is possible to know that introduction of permit's prices into the trading model causes the situation that similar amounts of permits are traded among countries, but permits expenditures are quite different, especially not profitable for the USA. This conclusion seem to be reasonable, because free market prices are a little bit higher than in the optimal Walrasian model. Of course results obtained using the old model (Tab. 3) with EA simulations are almost identical with these ones obtained by researchers using different optimization techniques.

It should be noticed that the final equilibrium price for the market is obtained as a consequence of small steps – transactions between market participants, not as in the traditional approach – a result of global calculation. The obtained results are different because the price does not depend only on the shadow price, but also on the difference between shadow prices of market participants and also on some "negotiation abilities", modeled as a random value. Thus, there are several local equilibrium points among countries and the market simulation stops when no profitable transaction can be done.

5 Conclusions

This article describes two applications of EA to market simulations. The first one is a non-standard method without traditional selection of individuals but very applicable for game problems. the second is more traditional version of EA, but with specialized operators for market simulations. As it could be noticed, evolutionary algorithms are very flexible tools for analyzing economic phenomena. It is possible to consider more different factors applying EA than using standard methods, for instance including prices of emission permits to ideal market. Introduction of several additional effects, like better models of price negotiations and uncertainty of reported emissions will be a challenge for continuing research in this domain.

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