Simple 3D object reconstruction using global optimization

Anna I. Okrasa, Tomasz M. Gwizdałła*

University of Łódź, Dept. of Solid State Physics
Pomorska 149/153, 90-236 Łódź, Poland
* corr.author, email: tomgwizd@uni.lodz.pl

Abstract

In the paper we deal with the problem of three-dimensional shape reconstruction when its two-dimensional projections from selected directions are available. This situation corresponds e.g. to the trial of digitalizing the shape from photographs. We propose a methods for creating the mesh, projecting the shape keeping the information about light and perspective. On this basis we use SA technique to modify this shape in a controlled way in order to reproduce the initially assumed one best.

1 Introduction

The problem of 3D shape from 2D silhouette reconstruction is now one of the most interesting and challenging problems of computers graphics algorithmics. There is a large number of papers which try to deal with this issue using different approaches. Among them one can mention eg.: volumetric intersections of polyhedra analysed analytically [2] as well as in the application to the hard problem of tree shape [11]; tomographic model which obviously needs the more specifically prepared data coming from the intersections [10, 1] used especially for medical purposes. Also a lot of specific algorithms based frequently on different methods of projecting the shape have risen, like Mercier's [9] one which needs the specific hardware or Chen's [3] where the points of observation are constant. An interesting model of continuous convex reconstruction from multiviev has been presented in [7, 6]. In the another group of papers one can find the use of global optimization methods to different visualization problems. Quite successful are eg. [8, 5] where the reconstruction doesn't concern the whole sample but only the surface or a little different problem of image quantisation [4].

In this paper we are going to present some preliminary works concerning the application of simulated annealing global optimization to the problem of 3D object reconstruction using the digitalization offered by OpenGL library. It may be noticed that the problem of back 2D picture estimation form proposed 3D shape is often studied using different methods of projections. In our method we use a rich algebraic solutions incorporated in this library to visualize the object as a view from the arbitrary position of arbitrary number of cameras and in arbitrary conditions. We propose the method of differentiating pictures in order to compare them and present some results for simple polyhedra.

2 Shape creation and picture rendering

Our reconstruction procedure has to start from the defining of initial shape which will further undergo the optimization. The method of construction is related to the method of projecting the shape onto two-dimensional pictures corresponding to the camera view. Because we are going to use OpenGL algebraic libraries the shape will be presented as constructed from flat triangles. The scheme of this process is shown on fig.1.

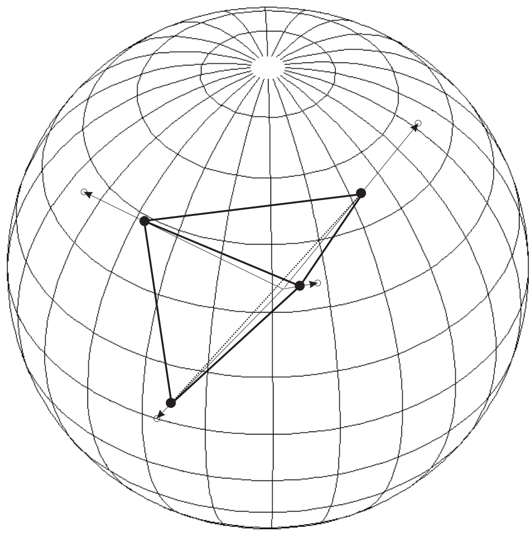

Figure 1. Schematic presentation of the figure creation method. Empty circles are sampled on the sphere, full circles are the internal points sampled on the line between the centre of sphere and the empty circle, solid lines correspond to the visible edges while the dashed one to the only invisible one.

Initially we define a spherical hull encompassing the reconstructed object. Then the appropriate number of points is sampled uniformly on the sphere. These points are represented on the figure as open circles. Selected points are connected with the centre of sphere and on these lines there are sampled real point, shown as black circles. Thin lines connect points on the sphere with its center while thick ones corresponds to the visible edges of exemplary tetrahedron. Thin dotted line shows an invisible edge. Such a method has certainly some advantages as well as drawbacks. Both of them are connected to the organization of points. From one point of view the spherical notation makes it possible to order the points according to their radial and azimuthal angles. From the other hand it is very time consuming to perform such orderings. The presented approach is also appropriate to code wide class of objects. This remark is obvious for all convex objects but also for those which do not obey this condition one can find the successful representation. The only requirement is that we have to find an internal point such that every half-line going from this point intersects the hull exactly one time.

The object created and represented according to above description is then projected as a view in all of the cameras defined in a system using OpenGL libraries. This algorithm allows to introduce into reconstruction procedure all features offered by this library. One can define, except of triangulated shape, the material covering the walls, refraction coefficients, position and character of lights sources. The number of cameras as well as light sources is practically unlimited, so generally we can define arbitrarily complicated system of 2D silhouettes creation.

3 Optimization

As an optimization procedure we use the well known simulated annealing (SA). The choice of this method seems to be almost natural for this case. Having well defined set of points, during every single step we try to shift the coordinates of only one of them. Contrary to the shape creation, we use here the Cartesian coordinates. In the further considerations we will denote: T_0-initial temperature, η_T-diminishing factor for temperature, x_0-initial value of sampling interval width, η_x-diminishing factor for interval width.

Two stop conditions were defined. We force the stop after given number of steps, usually it is 1500 or 2000 as it will be visible on plots. Alternatively we allow to stop when either current temperature or current interval width reaches selected small value. Usually, the process which finished due to this second condition stopped in the local minimum.

The final object which can be analysed during the optimization process is the set of $3 * n_{cameras}$ matrices. The numbers come from the need to present the view from the positions of all cameras (therefore the size of matrix is given by the product of width and height of the original picture obtained from this camera) as well as for three basic colours independently. The search space correspond to the possibility of filling of all single $pixel + colour$ combinations with the preciseness related to the assumed colour depth. In our calculations it was always equal to 256. The objective function used in minimization has to reflect the differences especially in color interpretation. Therefore we decided to define the difference between images img_1 and img_2 as:

$$f(img_1, img_2) = \sum_{cam} \sum_{pix} (|\Delta r| + |\Delta g| + |\Delta b|) \tag{1}$$

In the formula above the first summation is taken over all cameras, the second one over all pixels in sample and Δr, Δg, Δb means the relative difference of intensities of respectively red, green or blue color assigned to a pixel. With this formula the maximum difference between the white and the black pixel is equal to 3. At this stage we don't also take into account the weights connected to the sizes of pictures obtained using different cameras.

4 Exemplary results

We start the presentation of results from the most simple three dimensional convex shape which can be build from triangles, ie. tetrahedron. In the fig.2 there are presented the images obtained in different phases of optimization for the such a tetrahedron.

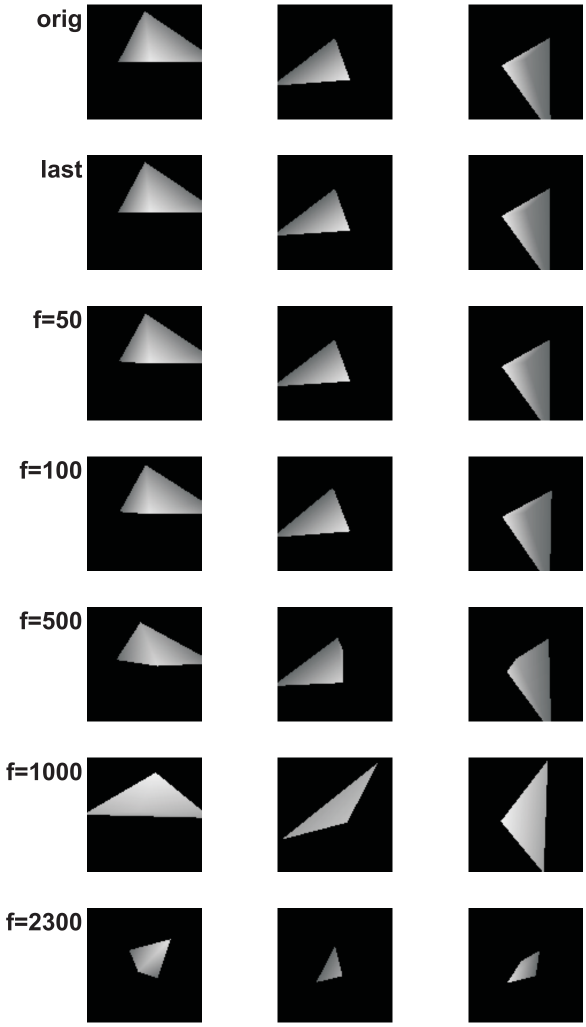

Figure 2. The images of optimized 4-vertex shape.

As it has been written earlier, the number of cameras is arbitrary. In all experiments we decided to use three cameras and the pictures are organized in the way that as well the original one those obtained during optimization process for the specific camera are grouped in the same columns. The rows contain images at different stages of optimization, namely: (a) original picture, (b) the last obtained during minimization ($f \approx 18$), (c) $f \approx 50$, (d) $f \approx 100$, (e) $f \approx 500$, (f) $f \approx 1000$, (g) starting configuration $f \approx 2300$. The image sizes are identical for all three cameras (100x100) and the maximal objective function value may be $9 * 10^4$. It should be pointed that the method of constructing of initial shape leads to quite reasonable values of difference for starting configuration.

First steps of optimization leads especially to enlarge the tetrahedron in order to preserve the correct order of magnitude and at last steps one can observe the convergence to the real shape. For the values $f < 100$ one can hardly observe differences between successive images. On the presented plot it can be seen only small shift of the lowest point which may be observed as a slight curvature on the left one where the border between the background and the enlightened part should be exactly horizontal. The final images ($f \approx 18$) may not be distinguished from the original one although some slight differences still occur.

In order to compare the results for another pattern in fig.3 there are presented the original and the best obtained images for different tetrahedron.

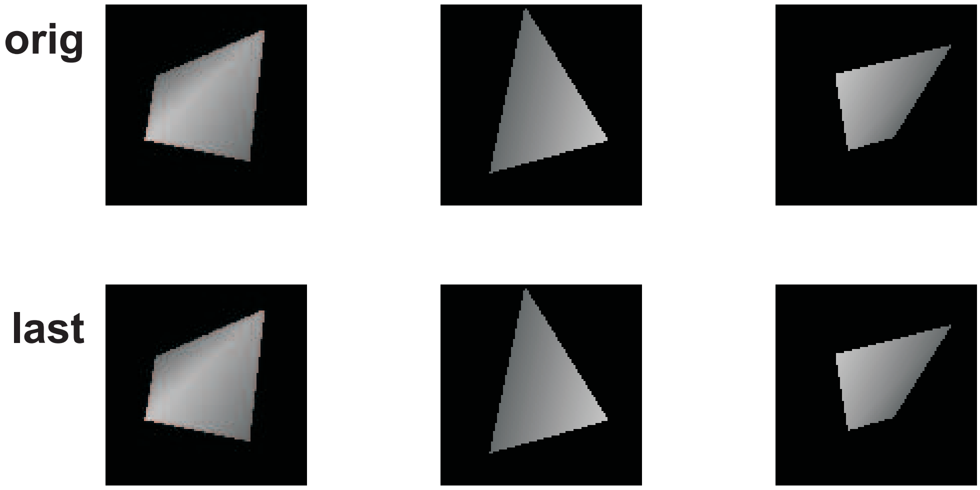

Figure 3. The original and the best images for another 4-vertex shape.

In the fig.4 there are shown the results of optimization for hexahedron. Here one can visibly distinguish the images especially due to insufficient light at the bottom ones. The final difference value is about 250 and one may expect that in next steps would be decreased. We want however to keep the stop condition at 2000 steps.

Let us now return to the first shape and present some efficiency results for this process. The crucial parameters are certainly those characteristic for simulated annealing, listed in the preceding section. In the table 1 there are presented main statistical values characterizing the optimization process for different sets of these parameters.

It may be discussed which one of them is the most important, whether we are interested in one good "hit" in a good result or in the larger set of better approximations for eg. further hybrid algorithm. It seems to be justifiable that the values of η_T and η_x about 0.95 which means rather slow (however not extremely) cooling are most successful.

We can try also to make an attempt to early qualification whether the optimization run is promising or it will probably fall into deceptive local minimum. In fig.5 we can observe the dependence of objective function on the number of step for 4 runs: two "good" ones and two "bad" ones.

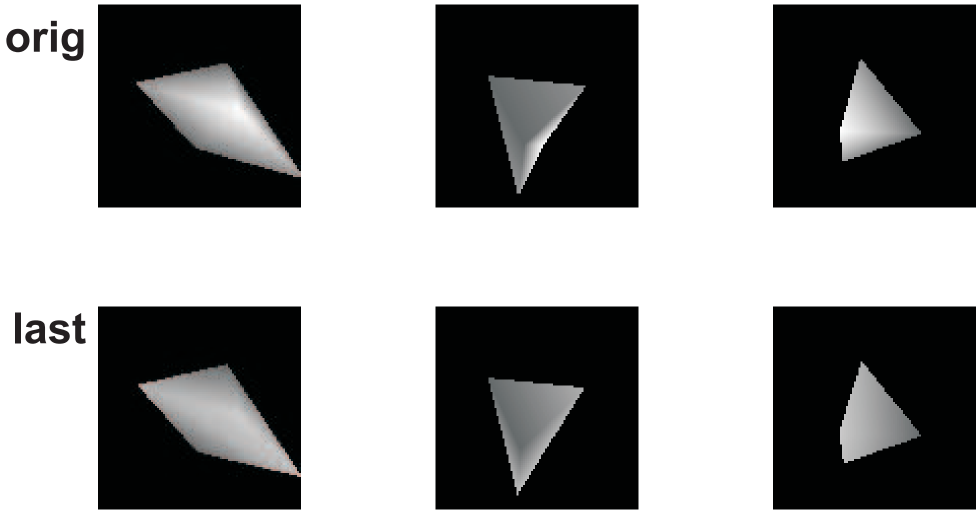

Figure 4. The original and the best images for 5-vertex shape.

T_0	η_T	x_0	η_x	average	deviation	median	min	max
100	0.95	4	0.95	325	398	42.9	7.7	974.3
100	0.95	4	0.97	259	387	24.9	9.8	871.2
100	0.97	4	0.95	1080	686	979	266.0	2746
100	0.97	4	0.97	505	414	621	42.7	1282
50	0.95	4	0.95	254	388	23.5	4.2	953.8
100	0.9	5	0.9	504	511	424	13.7	1372

Table 1. The statistical estimations for different optimization parameters. The numbers in columns concern the objective function values at the end of optimization process.

It may be observed that in initial phase the dependence is of exponential character (correlation coefficients are equal to 0.99, 0.89, 0.95, 0.87 for successive runs as enumerated in the figure) but the damping coefficient differ significantly. For the better runs it is about $4 * 10^{-3}$ while for the worse $1 * 10^{-3}$. This fact may give an interesting information about the predictability of efficiency.

5 Conclusions

Concluding the presentation we have to put an attention on few facts which partly has been mentioned in the text. Although the method seems to produce correct result, the samples which were taken into account were quite simple. The comparison of convergence for 4-vertex and 5-vertex shapes show that every additional point makes the optimization significantly harder. The positive information is however that even greater difference for hexahedron is of quantitative character and qualitatively we can say that we are generally

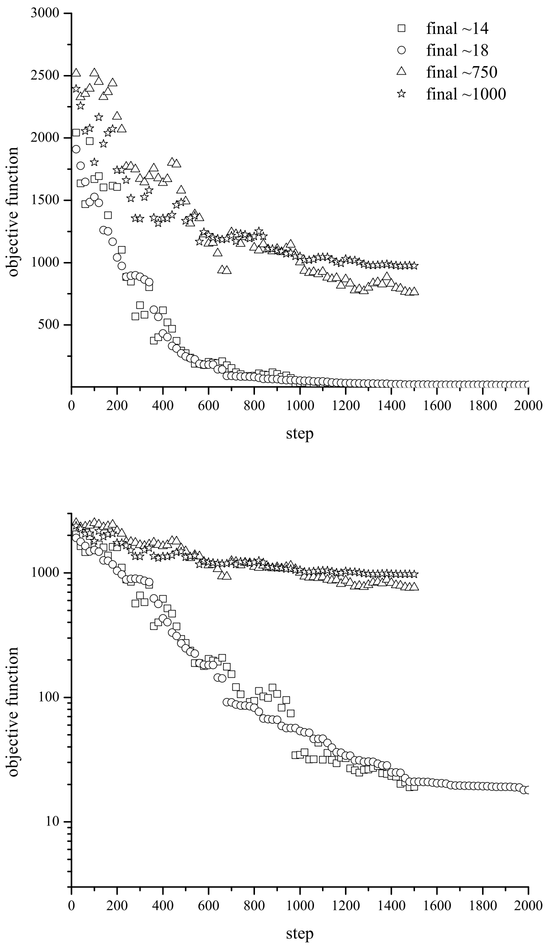

Figure 5. Dynamics of optimization process.

in the basin of attraction of global minimum. This observation leads to the conclusion that some form of local optimization should be proposed. The comparison of images (b)-(d) in fig. 2 shows that in this phase (obtained really after about thousand steps) SA works indeed only as a local optimizer.

It may be reasonable to use other global optimization techniques like eg. genetic algorithm. Here one should however notice the problem that will also arise for the samples with greater number of vertices. Using SA we can try to control the positions of points checking the fulfilment of conditions described earlier. The lack of this control may lead to the intersections of triangles what makes the analysed set of points useless. As a solution we are going to use Delaunay triangulation on a sphere which is unfortunately the next time consuming factor.

Finally we can say that the proposed attempt is promising although needs a lot of detailed improvements.

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