Influence of the Adaptive Saddle Slope Gradient on the Evolutionary Saddle Crossing Time

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Abstract The ability to cross saddles separating adaptive optima is one of the most interesting features of adaptive processes. It is especially promising in the case of multimodal optimization. This contribution deals with the impact of the saddle slope gradient on the time of the saddle crossing. A suitable adaptive landscape with variable saddle parameters is proposed. A simple model of evolution with the real valued traits; proportional selection, normally distributed modifications and without recombination is used.

1 Introduction

This contribution deals with the problem of the evolutionary saddle crossing. The adaptive landscape is formed of two adaptive peaks separated by a distinct adaptive saddle. An adaptive saddle can be defined as a part of the landscape that connects two adaptive hills, but the surface of the saddle is lower than both hills at the same time, one can imagine this as a mountain valley.

The process begins with the lower peak already populated. The population of individuals fluctuates on it due to the effect of proportional selection and normally distributed mutations of individuals. Finally fluctuations lead to saddle being crossed - an escape from local optima pitfall. Afterwards, exploitation of the higher adaptive peak improves the average fitness of the population.

The population drift through the saddle proceeds along saddle rim - individuals try to occupy the most uplifted area[2]. Search direction perpendicular to the saddle rim cause bigger quality loss than move along the saddle. Such conditions provide clear search direction toward one of the adaptive hills, thus significantly reduce random drift of an individual in the saddle vicinity.

Because of the random nature of individuals modification during reproduction phase, all search directions are equally possible so, on steep saddles (big slope gradient) a number of mutations leading to individual extinction might by significant. In few steps the mutation process can put an individual in the area with high quality degradation. On the other side low saddle slope gradient or small average value of the mutation standard deviation may trigger a random drift inside saddle area. In both described situations saddle crossing effectiveness is reduced. Therefore deeper insight into influence of the saddle slope gradient on the evolutionary saddle crossing process is worth of analysis. To perform experiments a proper adaptive landscape, with adjustable saddle slope gradient was introduced.

Earlier research deals with many aspects of the considered evolution model, such as: search space dimension, population size, mutation rate, adaptive landscape shape, impact of random perturbations. Those findings, regarding basic model configuration, are used as the reference point for experiments results considering the influence of the adaptive saddle slope gradient on the evolutionary saddle crossing time. This paper is organised as follows: the model of evolution is described in Section 2; an adaptive saddle and the landscape configuration are characterized in Section 3; the data on numerical experiments are presented in section 4; Section 5 presents and discusses simulation results; Section 6 concludes the paper.

2 Model of Evolution

The simple model of phenotypic, asexual evolution with no-overlapping generations is considered[4]. The population of m individuals evolves in the unbounded n-dimensional search space. The type of every individual x is given by a vector of its real valued traits $x = (x_1, x_2, \cdots, x_n)$, and the fitness value $q(x)$. The fitness function $q(.)$ generates the adaptive landscape. It is assumed that the landscape is multimodal and consists of adaptive hills separated by an adaptive saddles. Reproduction proceeds in two steps:

1. **Selection**. Parents for new generation individuals are selected, with probability proportional to the fitness value - proportional selection.
2. **Mutation**. Descendants inherit slightly modified parental traits. Each trait is mutated by adding a value of the normally distributed variable, with mean 0 and variance σ^2. The value of σ is small with regard to the linear dimensions of the adaptive landscape.

This simple model captures the essentials of the Darwinian evolution. The process is path dependent. In the beginning the individuals of initial population concentrates quickly into a cluster of types, with radius of about σ[2],[1]. Then this cluster moves toward more elevated area of the adaptive landscape. When population reach the top of the adaptive peak (local optima) enters the stage of the selection-mutation equilibrium. While the process of mutation tries to spread population and provides exploration factor, the proportional selection process keeps it in the vicinity of the peak by preferring individuals with higher fitness as parents for the next generation. Individuals moves randomly around the peak with the average fitness value distinctly lower than the locally optimal. The population might escape the local optima pitfall if there is a saddle that leads to some more attractive peak. The saddle crossing has a character of the random drift. The population moves along the saddle until more attractive area is found. When it happens, population again enters the stage of peak exploitation. The phase of saddle crossing and adaptive peak climbing are much shorter than the state of selection-mutation equilibrium[5].

Previous experience with the described model indicates that this is very efficient in saddle crossing, even when these saddles are tens of σ wide, especially if the populations are very small [5],[4]. Also this model keep its efficiency regardless of modification of the shape of the adaptive landscapes[3].

3 Landscape

The adaptive landscape consists of two adaptive peaks connected by an adaptive saddle. In previous research the landscape in major cases was constructed as a sum of two multidimensional Gaussian bells. Hover in presented experiments, adaptive peaks were modelled with multidimensional parabolas(1). Such approach was chosen to obtain a landscape with well-defined and easy to manipulate(in terms of shape modification) adaptive saddle, see Figure 1.

The adaptive saddle is defined by formula(2). Such function generates response surface with constant height value $\check{x}$, the slope gradient in dimensions orthogonal to saddle rim is controlled by coefficient s. The cross section of saddle for s values used in experiments is presented on Figure 2. Parabolas parameters for adaptive hills(1) were chosen to obtain profile similar to Gaussian bell used in previous research, see Figure 3. On Fig. 4 top projections of landscapes with slope gradient coefficient $s = 1$(Figure 4(a)), $s = 4$(Figure 4(b)) and $s = 16$(Figure 4(c)) are presented, the difference in saddle size in dimension $n = 2$ can be noticed.
The proposed fitness function(3) holds following assumptions: non negative fitness value, global optima peak twice as high as local one, distinct saddle lower than both of adaptive hills, symmetry along saddle rim plane.

$$h(x) = max\left(1 - 4 \cdot \sum_{i=1}^{i=n} x_i^n; 0\right) + 2 \cdot max\left[1 - 4 \cdot \left((1 - x_1)^2 + \sum_{i=2}^{i=n} x_i^2\right); 0\right] \quad (1)$$

$$r(x) = \check{x} - s \cdot \sum_{i=2}^{n} x_n^2 \qquad \text{s - slope} \quad (2)$$

$$q(x) = \begin{cases} h(x) & x_1 \leq 0 \\ max\Big[h(x); r(x)\Big] & 0 < x_1 < 1 \\ h(x) & x_1 \geq 1 \end{cases} \quad (3)$$

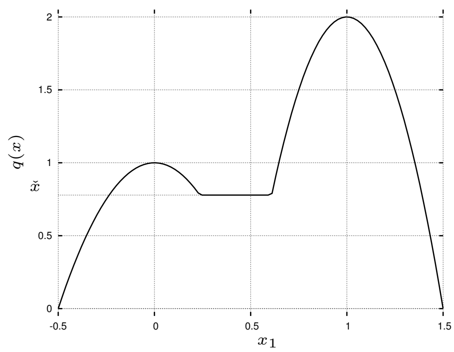


Figure 1. Landscape cross section, saddle slope coefficient $s = 4$, $\check{x}$ - saddle height.

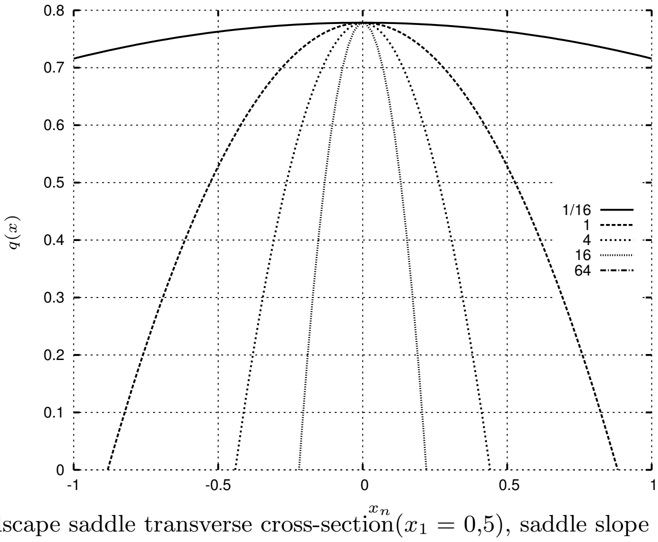


Figure 2. Landscape saddle transverse cross-section($x_1 = 0{,}5$), saddle slope coefficient $s = 1/16, 1, 4, 16, 64$, dimension $n = 2, 3, 4, \ldots$.

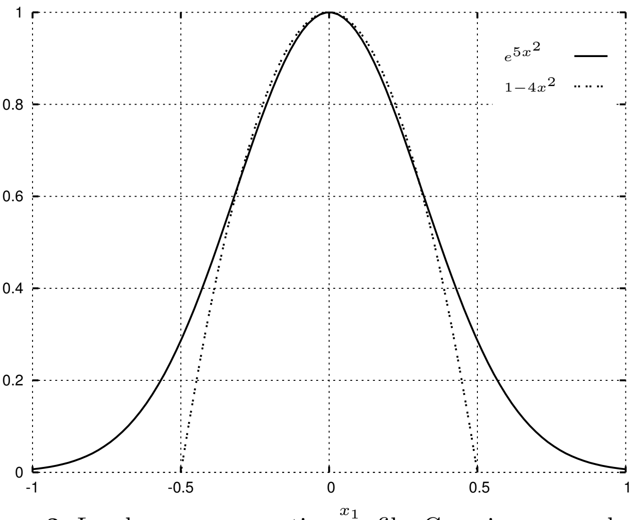
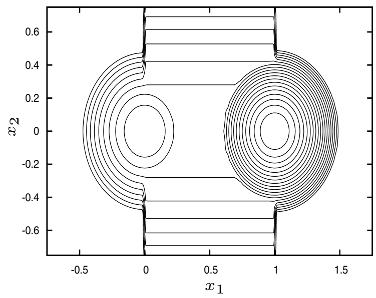


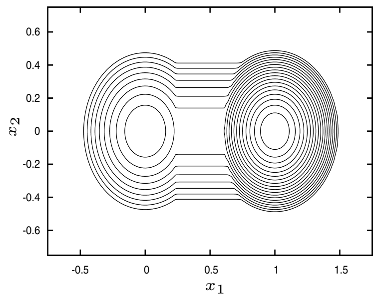
Figure 3. Landscape cross-section profile, Gaussian vs parabolic.

4 Numeric Simulations

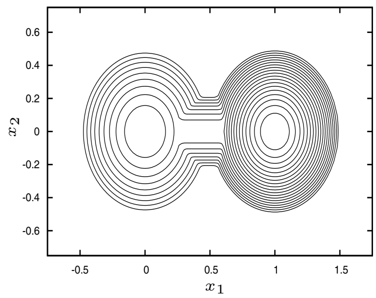
The experiments were designed to check the influence of saddle slope gradient on the adaptive saddle crossing time. Thus a set of fitness functions (2)(3) with different saddle slope gradients was used $s \in \{1/16, 1, 4, 16, 64\}$, all cases are shown on Figure 2. For set of basic evolutionary process parameters: n-search space dimension, m-population size, σ - mutation variance simulations were performed for each of five



(a) slope gradient coefficient $s = 1$



(b) slope gradient coefficient $s = 4$



(c) slope gradient coefficient $s = 16$

Figure 4. Landscape top projection.

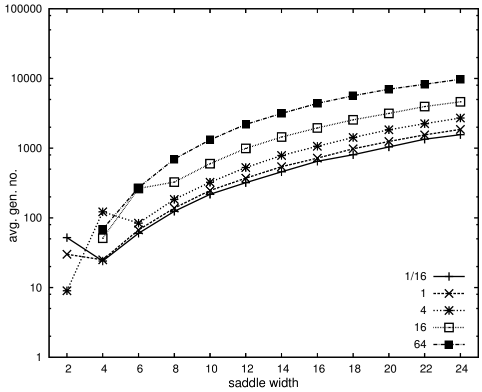
landscapes. The saddle crossing efficiency was measured as average time(in numbers of generation) required to obtain quality improvement, that is the generation number k for which the condition (4) is met.

$$q(x_i^k) \geq (\check{x}), i = 1, 2, \ldots n \tag{4}$$

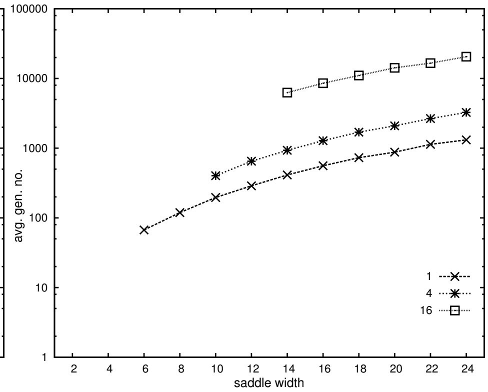
Such stop condition is very simple to check and quite intuitive. Although, especially for multidimensional problems, population might move in the global peak vicinity before quality improvement occurs[4]. The simulations were execute 2500 times for each parameters context, the result were then averaged. Because experiments are focused on the adaptive saddle, the saddle width is expressed relative to value of the mutation standard deviation σ, thus saddle k-width denotes that $w_s = k\sigma$.

5 Simulation Results

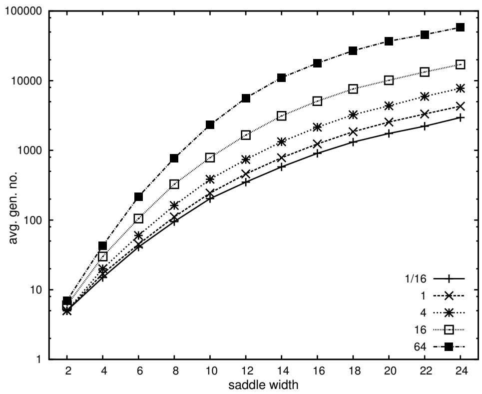
On Figure 5 plots with simulation results are presented. Missing points indicates, that for given simulation parameters stop condition(4) was not reached within 1000000 generations. As expected, in major cases moderate slope gradient decries average



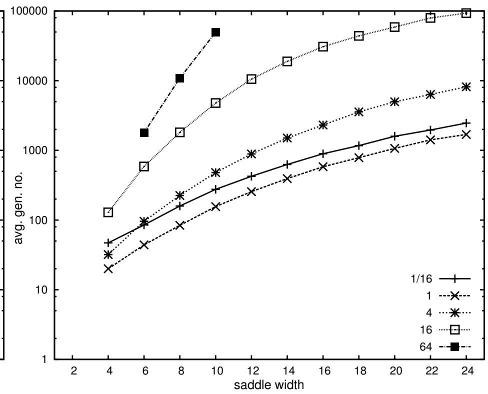
(a) space $n = 2$ pop. size $m = 4$



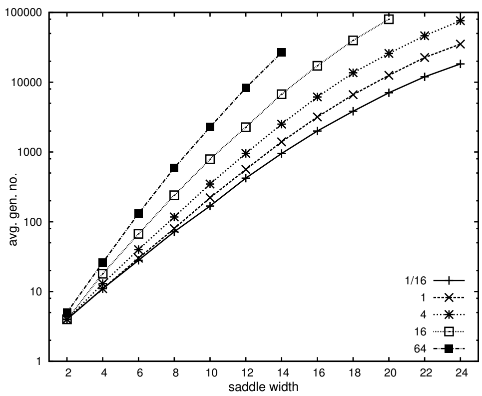
(b) space $n = 4$ pop. size $m = 4$



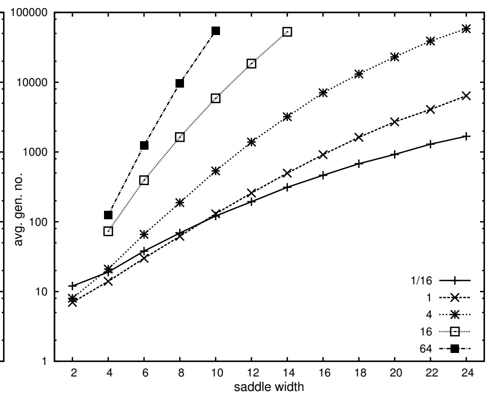
(c) space $n = 2$ pop. size $m = 8$



(d) space $n = 4$ pop. size $m = 8$



(e) space $n = 2$ pop. size $m = 16$



(f) space $n = 4$ pop. size $m = 16$

Figure 5. Average saddle cross time vs. saddle width, results averaged over 2500 runs. Search space dimension $n = 2, 4$, population size - $m = 4, 8, 16$, saddle relative width $w_s = 2, 4, 6, \ldots, 24$. Saddle slope parameter $s = 1/16, 1, 4, 16, 64$.

saddle cross time comparing to steep one. Although situation presented on Figure 5(d) shows that fine saddle slope may give opposite results $s = 1/16$ and $s = 1$. High cross time on steep saddles $s = 4, 16$ is caused by a "sudden death" effect. The area of the saddle is relatively small thus individuals after modification are placed in a region with low fitness value.

An interesting effect is observed for saddle with slope factor $s = 1/16, 1, 4$, saddle crossing time is lower for four dimensional space then for two, that effect increases with population growth. On the other hand for rough saddle slope configuration saddle crossing time growth is much bigger for four dimensional search space then two dimensional. Quality improvement, thus saddle pass over, was not achieved for small population(Figure 5(b)) and both extreme landscape configurations: $s = 1/16$ and $s = 64$. In first case low selection pressure(small population and moderate saddle shape) triggered random search. This effect can be also observed on Figure 5(d), saddle cross time for landscape with $s = 1/16$ is bigger then for one with $s = 1$. This outcome is reduced with population size increase - see Figure 5(f).

6 Conclusions

Presented results clearly shows that the local adaptive saddle configuration has a major influence on the evolutionary saddle crossing time. In this contribution the role of the saddle slope gradient was examined. The proposed method[4] keeps the ability to overcome local optimal pitfall on proposed, modified landscapes. This key feature of evolutionary algorithms is particularly valuable in solving multimodal optimisation problems[6].

The saddle slope gradient has a noticeable impact on saddle crossing process. Although the results shows, that simple assumption that moderate saddle slop will decrease saddle cross time can not be done.

The proposed landscape allows for easy manipulation of saddle slope configuration, yet the saddle rim shape can not be modified and it has no distinct minimal value point(contrary to the adaptive landscapes used in previous findings). Thus more research and new saddle configuration are planed to allow deeper analysis of the issue discussed in this contribution.

An interesting and worth of detailed examination is comparison of the presented evolution model efficiency on diverse landscapes configuration with other evolutionary algorithms, for example[7].

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