Evolutionary Multi-Objective Optimization of Laminates

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Abstract. The paper deals with the application of the global optimization methods to the multi-objective optimization of laminates. A multi-layered, fibre-reinforced and hybrid laminates are considered. Different optimization criteria connected with the laminates' cost, modal properties and stiffness are taken into account. As the optimization criteria usually cannot be satisfied simultaneously, the multi-objective optimization methods are employed. The multi-objective evolutionary algorithm with the Pareto approach is used as the global optimization method. The Finite Element Method is used to solve a boundary-value problem for the laminates. The stacking sequence and the number or plies made of different materials are design variables. The discrete as well as continuous fibre orientation angles in particular plies are considered. Numerical example presenting non-dominated solutions for two contradictory optimization criteria is attached.

1 Introduction

Multi-layered laminates are the fibre-reinforced composites built of a definite number of stacked and permanently joined plies (laminas). Among different types of composites, laminates have especially great strength/weight or elasticy/weight ratios with comparison to traditional, usually isotropic structural materials (e.g. steel or aluminium alloys [11]. Multi-layered laminates are typically composed of laminas with different fibres orientation, while the fibres in particular laminas are placed unidirectionally.

It is possible to influence or to design the laminates' properties by manipulating several parameters, like components material, fibres orientation, stacking sequence or layers thicknesses. As a result laminates are eagerly used in modern industry as high-efficient materials.

The cost of laminates usually rapidly increases with their strength. In order to find a balance between the laminate cost and its strength (or other required properties) laminates are composed of laminas made of different materials [2]. Such laminates are called the hybrid ones. There exist a few main types of hybrid laminates [10]: i) interply hybrid laminates having stacked two (or more) homogenous reinforcements; ii) intraply hybrid laminates with tows of constituent fibres mixed in the same ply; iii) intermingled hybrid laminates in which highly fibers of different kinds are randomly mixed; iv) selective placement hybrid laminates with additional reinforcement located in places, where additional strength is required; v) superhybrids composed of metal foils or metal composite plies stacked up in a given sequence and orientation. The interply hybrid laminates with plies composed of two different materials are considered in the present paper. The internal layers are built of a less expensive material having worse properties while the external layers are composed of a material with better properties but more expensive one. It is also assumed that laminates have a plane of symmetry (symmetrical laminates). As a result there is no coupling between shield and bending states ([8]) and e.g. there is not the bending effect during the tension, which is an important feature from the practical point of view.

The aim of the paper is to find the values of ply angles and the number of external plies for the interply hybrid laminate in order to satisfy more than one criterion. The criteria are connected with the laminate cost and other properties of the laminate, like modal properties and/or laminate stiffness. Such attitude leads to the multi-objective optimization task. In order to solve it, the global optimization method in a form of the multi-objective evolutionary algorithm is employed. The Finite Element Method (FEM) software is employed to solve the boundary-value problem for laminates.

The single-objective optimization problems for hybrid laminates were successfully solved in prior papers, e.g. [5], [6].

2 The Multi-Objective Optimization

A single-objective optimization task always leads to one optimal solution. In many practical engineering problems more than one objective function must be satisfied in the same time. The objectives may have opposing characteristics - decreasing the value of one of the functions may increase the value of another [9]. There exist many attitudes to the multi-objective optimization problems ([13]). In the present paper the multi-objective optimization problem is solved by means of the multi-objective evolutionary algorithm (MOEA)with the Pareto approach coupled with the FEM software.

The aim of the multi-objective optimization task is to find the optimal set of ply angles and the number of plies made of particular materials for given criteria. Solution of the problem is represented by more than one objective function with assumption, that optimization criteria are (or may be) contradictory. As a result it is not possible to improve all the criteria simultaneously and the aim of the optimization is to find solution with the values of all objective functions acceptable to the designer. This attitude leads to a set of the equal solutions of the problem

A MOO problem can be expressed as searching for the vector $\mathbf{x} \in \mathbf{D}$, where $\mathbf{D}$ is a set of admissible solutions being a subset of design space $\mathbf{X}$:

$$\mathbf{x} = [x_1,\ x_2,\ \dots\ x_n]^T \tag{1}$$

which minimizes the vector of k objective functions:

$$f(\mathbf{x}) = [f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ \dots\ ,\ f_k(\mathbf{x})]^T \tag{2}$$

It is required for the vector $\mathbf{x}$ to satisfy the m inequality constrains:

$$g_i(\mathbf{x}) \leq 0 \quad i = 1, 2, ..., m \tag{3}$$

and the p equality constrains:

$$h_i(\mathbf{x}) = 0 \quad i = 1, 2, ..., p \tag{4}$$

In the present paper the multi-objective optimization utilizes the Pareto optimality attitude [3]. A point $\mathbf{x}^* \in \mathbf{X}$, is called the Pareto-optimal or non-dominated one if and only if there does not exist another point $\mathbf{x} \in \mathbf{X}$ such that $\mathbf{F}(\mathbf{x}) \leqslant \mathbf{F}(\mathbf{x}^*)$, with at least one $F_i(\mathbf{x}) < F_i(\mathbf{x}^*)$ (for the minimization problems). The set of Pareto optimal solutions for an exemplary bi-objective optimization problem is presented in Figure 1 as a solid line. This set is called a Pareto front.

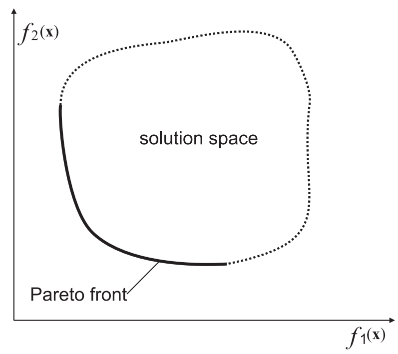

Figure 1. The Pareto front for the exemplary bi-objective problem

3 The Multi-Objective Evolutionary Algorithm

To increase the possibility of reaching the global optimum the global optimization methods [4] can be employed instead of local, typically gradient methods. Evolutionary Algorithms (EAs) are especially useful if: i) the information about the objective function gradient is hard (or even impossible) to obtain, ii) the objective function is multi-modal, which usually leads the gradient methods to the local optimum.

The only necessary information for the EA to work is the objective (fitness) function value. EAs process a set (population) of possible solutions and as a consequence the searching is multidirectional. Each possible solution is represented by a vector (chromosome) of design variables (genes).

In order to solve the multi-objective optimization problem the Non-dominated Sorting Genetic Algorithm (NSGA-II) [7] has been used. NSGA-II is more efficient than NSGA [12], it makes use of elitism and keeps diversity without specifying any additional parameters in comparison with the previous implementation. NSGA-II is based on several layers of classification of the individuals. In the main loop of the algorithm an initial, parent population (of size N) is randomly created. The population is sorted on the basis of the non-domination of individuals. Each solution is being assigned a rank value which is equal to its non-domination level (front 1 solutions is the best level, 2 is the next-best level, and so on). This classification of the individuals for minimization problem with two objective functions is presented in Figure 2.

In the first step of the algorithm the tournament selection, recombination (crossover) and mutation operators are used to create an offspring population. The tournament selection is employed as the mating selection method. This selection procedure takes into account both rank and the crowding distance and subsequently it allows searching for non-dominated regions and ensures the diversity of the population. To perform the selection of chromosomes the objective (fitness) function values have to be calculated. In order to calculate the values of the objective functions the boundary-value problem for laminates has to be solved. The commercial FEM software MSC.Nastran [1] has been used to solve the boundary-value problem for hybrid laminates.

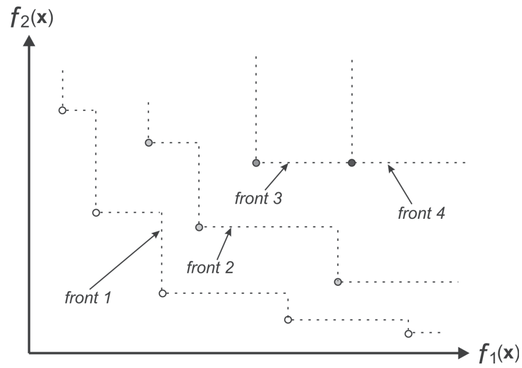

Figure 2. Classification of individuals in NSGA-II

Two evolutionary operators are employed. The simple crossover operator creates two offspring chromosomes from two randomly selected parents. Offsprings are composed of parts of the parents by cutting them in a random position and interchanging parts between them. The uniform mutation operator replaces a randomly chosen gene of the chromosome by the new value, which is the random value from the variable range with uniform distribution.

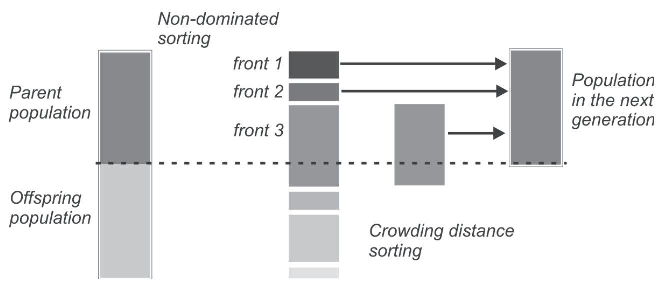

Figure 3. NSGA-II diagramm

Next a combined population containing current population and previously found best non-dominated solutions is formed. This combined population (of size $2N$) guarantees elitism in consecutive steps of algorithm. Subsequently, the non-dominated sorting procedure is performed on whole population (of size $2N$). Classification of the non-dominated individuals has the same form as in the initial population. The best solutions in i-th level are emphasized more than solutions in i+1 level. To choose exactly N population members crowding distance sorting procedure is applied. Considering two solutions with differing non-domination ranks the solution with lower (better) rank is preferred. In the case when both solutions belong to the same front the solution located in a lesser crowded region is preferred. The NSGA-II diagram is shown in Figure 3.

4 Optimization criteria

A fibre-reinforced hybrid laminates made of two materials are considered. The laminate is the symmetric one. The total number of laminas and ply thicknesses are assumed constant. The ply orientations (fibre ply angles) and the number of external plies of the hybrid laminate are the design variables. The ply orientations of laminates are usually limited to a small set of discrete angles due to the manufacturing process. There exist tow placements machines able to produce laminates with arbitrary ply angles, but they are not very popular due to their cost. In the present paper discrete and continuous variants of the optimization tasks are taken into considerations. The number of design variables is equal to a half of the plies number (due to the symmetry) plus one representing the number of external plies.

The objective functionals for multi-objective optimization of hybrid laminates can be defined as:

- The minimization of the structure cost.
  It is assumed that the thicknesses of laminas h_i, the number of plies N and areas of the plate A_i are fixed. Consequently, the cost of the laminate depends only on the number of plies made of each of materials and can be treated as varying in a discrete way. The dimensionless cost C of a laminate can be calculated as follows:

$$C = [n_e c_e + (N - n_e)c_i]\, h_i A_i \tag{5}$$

  where: n_e - the number of external plies; c_e - the unit cost of the external plies material $[1/m^3]$; c_i - the unit cost of the internal plies material $[1/m^3]$.
- The optimization of the modal properties of laminates in two forms:
  1. The maximization of the first eigenfrequency:

$$arg\max\{\omega_1(\mathbf{x}); \mathbf{x} \in \mathbf{D}\} \tag{6}$$

  2. The maximization of the distance between two adjacent eigenfrequencies:

$$arg\max\{\omega_i(\mathbf{x}) - \omega_{i-1}(\mathbf{x}); \mathbf{x} \in \mathbf{D}\} \tag{7}$$

5 Numerical Example

The aim of the multi-objective optimization is to find the optimal number of external plies and optimum values of ply angles in all laminas to satisfy two contradictory criteria:
a) minimize the cost of the structure - Eq. (5);
b) maximize the gap between 1^{st} and 2^{nd} eigenfrequencies - Eq. (7).

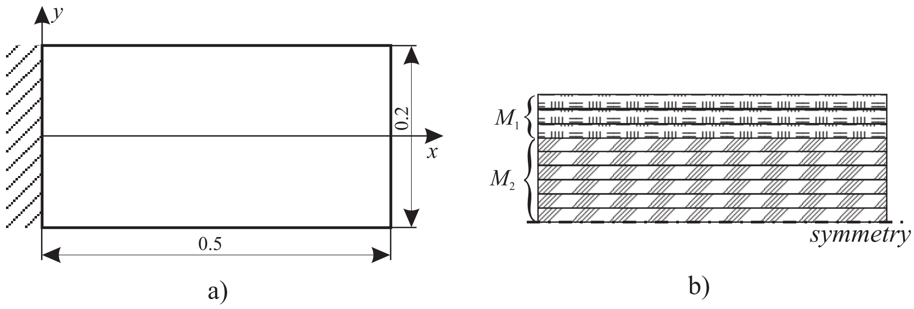

Figure 4. The laminate plate: a) shape and dimensions; b) exemplary materials location.

A symmetric rectangular hybrid laminate plate made up of 18 plies (Figure 4a) is considered. All plies have the same thickness h=0.0002m. To solve the boundary-value problem for the plate it is divided into 200 4-node (QUAD4) plane finite elements.

The external plies of the laminate are made of material M_1, the core plies are made of the material M_2 (Figure 4b). The number of the external plies is one of the design variables and can vary from 0 (simple laminate made of weaker material) to half times the number of the plies (simple laminate made of stronger material) due to symmetry. The material properties and unit costs are collected in Table 1. The parameters of the multi-objective EA are: the population size: $p_s = 50$; the number of genes in each chromosome: $n_g = 10$; the uniform mutation probability: $p_{um} = 0.1$; the simple crossover probability: $p_{sc} = 0.8$; the number of generations, $gen = 100$.

Table 1. The hybrid laminate - material parameters

Material	E_1 [GPa]	E_2 [GPa]	ν_{12}	G_{12} [GPa]	ρ [kg/m^3]	unit cost [$1/(m^3)$]
M_1	181	10.3	0.28	7.17	1600	6.0
M_2	38.6	8.27	0.26	4.14	1800	1.0

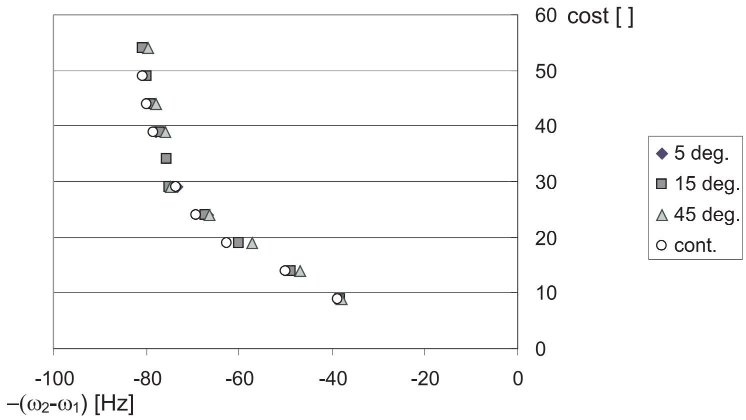

Figure 5. Numerical example - optimization results.

Each ply angle could vary in the range of $< -90^o; 90^o >$ in a discrete way (every 4^o, 15^o or 45^o) or continuously, depending on the optimization variant. The results of the optimization are shown in the form of Pareto front in Figure 5. The values of design variables in form of the ply angles and the number of external plies for the optimal results are presented in Table 2. As the laminate is symmetrical, all results are referred to one half of it.

Table 2. Numerical example - design variables values for optimal solutions

case	solution no.	stacking sequence	no of ext. plies	$\omega_2 - \omega_1$ [Hz]	cost [ ]
cont.	1	(45/-43/29/-39/-29/42/-32/-17/53)s	8	80.716	49
	2	(41/-43/28/-30/-25/-17/29/-45/52)s	7	79.871	44
	3	(41/-44/28/-29/-26/-14/33/-73/48)s	6	78.518	39
	4	(-37/40/24/-37/-15/-24/-31/-19/54)s	4	73.539	29
	5	(-35/41/27/-29/-25/-19/-35/-74/-24)s	3	69.179	24
	6	(-37/30/51/-28/53/42/28/-8/53)s	2	62.741	19
	7	(-40/32/27/24/47/30/25/-9/43)s	1	49.980	14
	8	(45/-43/29/-39/-29/42/-32/-17/53)s	0	38.679	9
5^o	1	(-35/30/50/-25/15/-35/15/0/40)s	6	78.08	39
	2	(-35/25/45/-25/55/-20/15/-20/55)s	4	73.304	29
	3	(-35/25/45/-25/20/-35/15/-15/45)s	3	66.578	24
15^o	1	(-30/30/45/-15/0/45/0/0/60)s	9	80.627	54
	2	(-30/30/45/15/0/45/0/0/60)s	8	80.021	49
	3	(-30/30/45/-15/0/45/0/0/75)s	7	79.028	44
	4	(-30/30/45/-15/0/30/0/0/75)s	6	77.142	39
	5	(-30/30/45/-15/0/15/0/0/60)s	5	75.654	34
	6	(-30/30/45/-15/0/0/-30/-15/45)s	4	74.955	29
	7	(-15/30/45/-45/-15/30/-30/-30/45)s	3	67.120	24
	8	(-15/30/45/-45/45/45/-30/0/45)s	2	59.928	19
	9	(-30/30/45/15/0/30/0/0/30)s	1	48.716	14
	10	(-30/45/45/-15/0/30/-30/-30/30)s	0	38.118	9
45^o	1	(0/45/45/90/0/90/0/0/45)s	8	79.580	54
	2	(0/45/45/90/0/45/0/0/45)s	7	77.824	44
	3	(-45/45/45/90/0/0/0/0/45)s	6	75.858	39
	4	(0/45/45/45/0/90/0/0/45)s	3	74.678	29
	5	(0/45/45/45/90/90/0/0/45)s	1	66.496	24
	6	(-45/45/45/45/0/0/0/0/45)s	1	57.279	19
	7	(45/45/-45/0/0/0/0/0/45)s	1	46.791	14
	8	(0/90/90/0/0/90/0/0/45)s	1	38.037	9

6 Final Conclusions

Hybrid laminates are typically used to find a balance between cost and another properties of the structure. The stacking sequence optimization gives the possibility to obtain the required properties of the laminates. To satisfy different and contradictory criteria the multi-objective optimization has been used.

To avoid problems with calculation of the fitness function gradient the multi-objective evolutionary algorithm (NSGA-II) has been employed as the optimization method.To solve the boundary-value problem for laminates the finite element method has been employed. As the total number of laminas and their thicknesses was assumed to be constant, the cost of the hybrid laminate was discrete. The plies orientation angles were considered as continuous as well as discrete variables. Optimization results were presented in the form of non-dominated solutions (Pareto front).

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