Hybrid Optimization Algorithm using Evolutionary Algorithm and Local Search Method with its Application in Digital FIR Filter Design

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Abstract. This paper is devoted to the application of an Evolutionary Algorithm to the design of Finite Impulse Response filters (FIR). A hybrid algorithm is proposed, which consists of a robust global optimization method (Evolutionary Algorithm — EA) and a good local optimization method (Quasi-Newton — QN). An experimental comparison of the hybrid algorithm against EA and QN alone indicates that EA yields filters with the better amplitude characteristics than QN. Furthermore the hybrid method yields filters with even better amplitude characteristics and in some times, it needs significantly less time than EA alone to reach good solutions.

1 Introduction

Problem of matching numerical optimization methods to problems to be solved occurs in many engineering applications. In this paper we concentrate on two different numerical optimization methods: robust global optimization method (Evolutionary Algorithm - EA) and a well-known local optimization method (Quasi-Newton - QN). Then we propose to use a hybrid method [8, 11] which applies both EA and QN at one run. The hybrid method make good synergy of exploration and exploitation abilities (introduced by the EA and QN respectively).

The research was focused on a real engineering optimization problem: designing of digital FIR filters, which occurs in a processing line of a radar signal [6, 16]. Those digital filters are a part of a filter bank, which is needed for dividing signal into sub-bands which correspond to different velocities of detected object. The problem of designing filter bank which consists of k filters may be decomposed into k subproblems of designing a single filter with certain requirements. The basic one is that each filter's pass-band is separated from the pass-band of every other filter. Requirements for each filter in the bank are generally dependent on the function the filter should provide, e.g. passing or stopping signals of required frequencies.

Digital FIR filter yields its output as a linear combination of the delayed input signal. Therefore, the filter is completely characterized by the set of parameters — coefficient of this linear combination. In the digital signal processing domain, it is customary to analyze filter properties using the frequency characteristic $H(z)|_{z=e^{j\theta}}$ and, in particular, the amplitude part of that characteristic $A(z) = |H(z)|$. Typically [1], requirements for the filter characteristics are provided in a form of limits on the amplitude characteristic $A(z)$ values for different values od z. In contrast to the basic design method, where only a single attenuation level in the stop-band is specified, we consider several different values of the attenuation level in different parts of the stop-band. This requirement is needed to eliminate clutter[6] — objects we do not want do detect, like tree leafs, clouds, sea waves, etc. In radar problems we additionally need to perform in real time all computations necessary for signal processing, and thus the filter order should not exceed a certain value L. The value L comes from the time of observing one azimuth by the radar with the assumed sampling frequency.

When we consider the aforementioned assumptions in the filter design process, a L-dimensional, multimodal objective function is obtained. In this case we observed (see results in Section 4) that both EA and QN method alone are insufficient to obtain acceptable solutions in acceptable time.

This paper is aimed at testing the improvement comes from using the hybrid method in contrast to the global and local optimization separately in the FIR filter design task. The hybrid method is compared to different optimization methods whose representatives are evolutionary method [2, 13] and Quasi-Newton method [15, 4, 9].

The paper is organized in the following way. The optimization methods are described in Section 2. In Section 3 we describe filter design problem as an optimization task. In Section 4 we define the objective function and we give detailed assumption for filters parameters. Then, we present results of numerical experiment. Summary and outlook are provided in Section 5.

2 Optimization methods

2.1 Evolutionary algorithm

Evolutionary algorithm (EA) [2, 13] is a stochastic optimization method. In each iteration t, the algorithm maintains a population P^t that contains μ points from the search space. EA usually starts from randomly generated solutions, then it changes this proposed solutions in consecutive iterations. The changes depend on the quality of solutions in previous iteration. In the selection procedure candidates for the next generation are chosen. Probability of choosing a candidate is depends on its objective function value ("better" candidates are selected with higher probability), and the selection process is repeated to compute the next population. All selected points are changed using operations of mutation and crossover, which are named by biological evolution. Then the changed point become the population for the next iteration. More details on EAs can be found e.g. in [2, 13].

2.2 Quasi-Newton method

Quasi-Newton or, in the other words, variable metric method [4] is a local minimization method of a differentiable objective function. The method state is a single point x_t in search space. In each iteration, a direction d_t is established:

$$d_t = -{H_t}^{-1} \cdot \nabla f(x_t) \tag{1}$$

where $\nabla f(x_t)$ is a gradient of the objective function, and H_t is the approximation of the Hessian matrix. Then, a new point x^t is computed as a result of the line search, i.e.:

$$x_t = \arg\min_{\xi} f(x_t + \xi d_t) \tag{2}$$

After that, a new approximation of the Hessian is computed according to the formula [3, 5, 7, 15]:

$$H_{t+1} = H_t + \frac{\eta_t \eta_t^T}{\eta_t^T s_t} - \frac{H_t^T s_t^T s_t H_k}{s_t^T H_t s_t} \tag{3}$$

where $s_t = \mathbf{x}_{t+1} - \mathbf{x}_t$ and $\eta_t = \nabla f(\mathbf{x}_{t+1}) - \nabla f(\mathbf{x}_t)$

The method terminates if the gradient norm falls below a certain level. The QN method is designed for finding local minima only, but still it is a fast optimization method.

2.3 Hybrid method

According to [8, 11, 12] a significant improvement to the quality of solutions generated by the global optimization methods can be attained by making a hybrid algorithm in which a solution in finally tuned by local optimization methods. In this paper we consider the hybrid method which uses the QN method to improve solution obtained by EA when it reached the stopping criteria. Thus two algorithm are started one by another. It is expected that the global method (EA) reaches the neighborhood of the global extremum. Than the local method (QN) for exploiting this neighborhood is started. It is important to properly choose the stopping criteria for the global method. In this paper this assume is investigated in more details, and the result is presented in Table 3.

3 FIR Filter Design as an optimization problem

Well known filter design techniques [1, 14] are: the window method, the impulse response based method, and the equiripple design. In this paper FIR filter design task is described as an optimization problem.

In the formulation of the digital FIR filter design as an optimization task we search for a set b of complex filter parameters, so that a certain objective function, being the error of approximating $H_s(z)$ by $H_b(z)$, is minimized. The frequency function of the designed filter $H_b(z)|_{z=e^{j\theta}}$ is given by the formula:

$$H_b(e^{j\theta}) = \sum_{m=0}^{L} b[m] \cdot e^{-jm\theta} \tag{4}$$

$H_b(z)$ can be also written in a polynomial form with coefficients b_m for $m = 0, 1, 2, \ldots, L$:

$$H_b(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_L z^{-(L)} \tag{5}$$

Then the equation (5) can be transformed into:

$$H_b(z) = b_L(z^{-1} - z_0)(z^{-1} - z_1) \ldots (z^{-1} - z_{L-1}) \tag{6}$$

where z_m for $m = 0, 1, 2, \ldots, L-1$ are the zeros of the polynomial (5).

Each z_m is a complex number, and in our case we assume that all zeros z_m are located lied on the unit circle. This assumption provides maximum attenuation for each frequency where the filter's zero exist. Therefore, modulus of each z_m equals 1 and each z_m value can be identified with an angle ϕ_m in the Re/Im plane. Thus, the parameters to be optimized are real numbers from the range $(0 \ldots 2\pi)$, and their number equals the filter order L. We also assumed that $\max_\theta |H(e^{j\theta})|_{dB} = 0[dB]$, so the parameter b_L in (6) can be set to satisfy this requirement.

In the filter design we are interested in the amplitude of the frequency function only: $A(e^{j\theta}) = |H(e^{j\theta})|$. We assumed a linear phase characteristic, which is guaranteed by the placement of all zeros of the filter on the unit circle.

The objective function is the difference between the amplitude characteristic $A_s(e^{j\theta})$ (the goal characteristic) and $A_\phi(e^{j\theta})$ (filter characteristic defined by the vector ϕ). We compute the error:

$$e(\phi) = \left\| A_s(e^{j\theta}) - A_\phi(e^{j\theta}) \right\|_k \tag{7}$$

where $\|\cdot\|_k$ is the norm of the function which can be defined by:

$$\left\| A(e^{j\theta}) \right\|_k = \left[\int_0^{2\pi} |A(e^{j\theta})|^k d\theta \right]^{1/k} \text{ for } 0 < k < \infty$$

or (8)

$$\left\| A(e^{j\theta}) \right\|_\infty = \max_{\theta \in [0,2\pi]} |A(e^{j\theta})|$$

In practice we estimate (8) by an approximate sum, getting:

$$\left\| A(e^{j\theta}) \right\|_k = \left[\frac{1}{N} \sum_{n=1}^{N} |A(e^{j\theta_n})|^k \right]^{1/k} \text{ for } 0 < k < \infty$$

or (9)

$$\left\| A(e^{j\theta}) \right\|_\infty = \max_{n=1 \ldots N} |A(e^{j\theta_n})|$$

where $\{\theta_1, \ldots, \theta_N\}$ is the set of the frequencies dependent on the filter design criteria which is discussed in more details in Section 4.

Note that we are dealing with an unconstrained optimization problem, as the function (4) is a periodical one, since $e(\phi) = e(2\pi k + \phi)$, where k is a vector of L integer numbers. Additionally, for $\phi \in [0, 2\pi]^L$, the function (4) has many local maxima with respect to ϕ.

4 Numerical example

In Section 3 the FIR filter design is defined as a task of optimizing a specific objective function. In the numerical experiments, several further assumptions were made, which are discussed in following subsections.

4.1 Objective function and filter parameters

The definition of the objective function (9) requires the set of frequencies to compute the error. These frequencies were chosen in the following way (see also the discussion of

Fig. 1). θ_1 and θ_2 define the range of the pass-band. θ_0 and $2\pi - \theta_0$ are frequencies which limit the strong attenuation sub-band surrounding the $\theta = 0$. The remaining frequencies θ_n are defined as frequencies where $A_\phi(e^{j\theta})$ has maxima in the stop-band outside of the pass-band and the range $[0, \theta_0]$ and $[2\pi - \theta_0, 2\pi]$.

In figures below, the amplitude characteristic $A_\phi(e^{j\theta})$ of solutions to the FIR design filter is drawn in the logarithmic scale (decibel).

In Fig. 1, an example filter amplitude characteristic for $L = 8$ is shown. The filter pass-band is defined as a range of frequencies $[\theta_1, \theta_2]$ where the amplitude characteristic value $A_{12} = A_\phi(e^{j\theta_1}) = A_\phi(e^{j\theta_2})$ is $2dB$ smaller than at the maximum of $A_\phi(e^{j\theta})$ (depend on the design requirements). The value A_{12} for both θ_1 θ_2 is depicted as triangles. Two attenuation levels in the stop-band can be observed. The first group of frequencies θ_n (corresponding to the first attenuation level A_p) is shown as stars, and they are defined as maxima of each stop-bands ripple. In simulations, an approximation $\hat{\theta}_n$ to frequencies θ_n is used. This approximation equals $\hat{\theta}_n = \frac{\phi_{n-1} - \phi_n}{2}$, where ϕ_{n-1} and ϕ_n are two values which are closest to zero. The second attenuation level for frequencies from the range $[0, \theta_0]$ and $[2\pi - \theta_0, 2\pi]$ is defined by the A_0 value and in Fig. 1 $A_p = -30[dB]$.

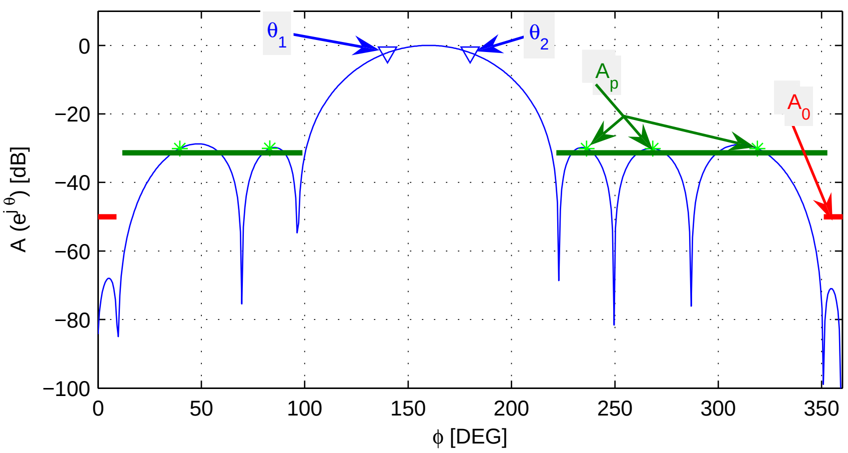

Figure 1. The amplitude characteristic $A_\phi(e^{j\theta})$ of example filter.

Before the filter design process starts, it is necessary to specify the following requirements: L, θ_0, θ_1, θ_2, A_{12}, A_p, A_0. Additionally, for the purpose of optimization we define the value of k, that is the norm coefficient used for computing (9). After the design process terminates we get the solution — the L frequency values corresponding to zeros z_m of the designed filter.

In some cases, it is possible to fix 1 up to 3 zeros in certain positions to force maximum possible attenuation in these frequencies. Thus, it is possible to reduce the number of variables by 1 up to 3.

4.2 Optimization methods

All simulations was performed under Matlab 7.0 (R14) environment. We used the following toolboxes: Genetic Algorithm Direct Search Toolbox (ver. 1.0.1), Optimization Toolbox (ver. 3.0) and Signal Processing Toolbox (ver. 6.2).

Quasi-Newton line search *fminunc()* function was used from Optimization Toolbox with the following options: LargeScale='off', HessUpdate='bfgs'.

Evolutionary Algorithm *ga()* function was used from Genetic Algorithm Direct Search Toolbox. We performed tests using EA with mutation and crossover in R^L space. The population size was $\mu = 20$. The mutation operator has the symmetrical gaussian distribution with the variance 1.0. The crossover operator implements the uniform crossover. It creates a random binary vector of an length L. Then it selects genes where the vector is a 1 from the first parent, and the genes where the vector is a 0 from the second parent, and combines the genes to form the child. EA uses the tournament selection (with size 4) and elitist reproduction (with elite count $\eta = 2$). The next generation consists of η individuals from previous generation (according to elite scheme) and $0.5(\mu - \eta)$ individuals produced by mutation and also the same number of individuals produced by crossover.

Hybrid method In this case we combine both global and local optimization method. First we start EA for certain numbers of generations to find the neighborhood of the global minimum, and next we perform the local method to find this minimum with better accuracy.

4.3 Example of filter design

In this paper we consider the design of a FIR filter defined by the requirements given in Tab. 1. An example amplitude characteristic of a filter which meets the design specification is given in Fig. 1.

Table 1. Filter design criteria

Parameter name	L	θ_0	θ_1	θ_2	A_{12}	A_p	A_0	k
Unit	-	DEG	DEG	DEG	dB	dB	dB	-
Value	8	9	140	180	-2	-30	-50	4

The filter has two attenuation levels outside of the pass-band ($\theta \in [140, 180]$). The first attenuation value is -30[dB] and it is required that all ripples should have similar amplitude. The second attenuation value is -50 [dB] it is required to stop signal at frequencies $\theta \in [0, 9]$ and $\theta \in [351, 360]$.

4.4 Simulation details

Several types of simulations were performed. The aim was to compare solutions (the objective function value) and the performance (the number of the objective function evaluations) for different optimizations methods. Both: solutions and performance were computed as mean value and standard deviation obtained from 100 independent runs of each method.

Window method This is a well known design technique [1, 14], which produces pass-band filter with one attenuation level at $-30[dB]$. This method is deterministic method only so a single run of the algorithm was performed.

Random method Vector filter's zeros are generated randomly with uniform distribution from the range $[0, 2\pi]$. 500 independent vectors are generated, and the best result is chosen as the solution. Note that the number of the objective function evaluations is also equal 500.

Quasi-Newton The QN method is applied to a randomly generated starting point. The starting point is generated with the uniform distribution from the range $[0, 2\pi]$

EA (25) The evolutionary algorithm is used, and the initial population contains points generated with an uniform distribution from the range $[0, 2\pi_0]$. In this case, 25 iteration were performed, so the number of the objective function evaluations is equals 500. Note that this numbers equals the mean number of the objective function evaluations in the Quasi-Newton method (Tab. 2).

EA (500) In this case, 500 iterations of EA were performed (10, 000 objective function evaluations).

EA (∞) In this case EA was terminated after getting a solution with the objective function value no greater than 0.5, so we measured the number of iterations when EA reaches the required objective function value.

Hybrid(i) = EA (i) + Quasi-Newton In this case, i iterations of EA were performed, and then QN method was used to improve the best solution found by the EA.

4.5 Results

In Tab. 2 the results are presented. We provide the objective function of the solutions, the number of objective function evaluations, and the time of the optimization process. For the window method, only the mean value of the solution is reported, because this method is deterministic and it yields always the same result. This method needs less than 1 second to compute result. The random method which generates 500 random points yields better result than the window method.

In our experiments the random solution (obtained form 1 runs) was used as the start point of optimization for both QN and EA.

Table 2. Results for different methods of optimization

	solution		numbers of function evaluation		time	
	mean	std	mean	std	mean	std
window	39.30	n/a	n/a		$< 1s$	n/a
random (500)	26.49	5.12	500	0.00	$1.03s$	0.01
QN	20.24	15.41	508	231	1.51s	0.58s.
EA (25)	19.52	4.95	500	0.00	1.19s.	0.02s.
EA (500)	10.51	2.99	$10,000$	0.00	24.0s.	0.33s.
EA (∞)	0.51	n/a	$2,142,740$	n/a	$3-5h$	n/a
Hybrid (500)	3.99	3.38	$10,342$	136	24.86s.	0.44s.

The Quasi-Newton method provides better mean solutions than both the window method and the random method. On average, QN needs about 500 objective function evaluations. EA with the stop criterion at 25 iterations (500 numbers of function evaluations) yields solutions similar to QN, but the optimization time is shorter by 0.3 seconds.

The EA(500) needs 24 seconds to yield solutions of the average objective function value equal to 10.51.

Further increasing of the iteration number in EA leads to unacceptable optimization time. EA(∞) needs more than $2,000,000$ iterations and a couple of hours to reach the solution with the objective function value equal 0.5.

Based on the previous results we proposed a hybrid method based on EA(500) and the QN. Although the computation time increases from 24 second up to 24.86 seconds, but when comparing to EA(500), the objective function value decreases from 10.51 down to 3.99. The best solution was 0.31. We observe that the distribution of solutions is a multimodal function (see histograms of solution in Fig. 2). This indicates that the objective function has at least two local minima, with large attraction basins.

We performed tests to verify the influence of the number of EA iterations on the quality of solutions yielded by the hybrid (EA + QN) approach. The result are given in Tab. 3.

From the results it evidences that there is a tradeoff between the computation time and the quality of results. Anyway, it is always much more adviceable to use the hybrid approach instead of any other methods used in this comparison.

5 Conclusions

In this paper we study the hybrid method based on evolutionary algorithm joined with an efficient local optimization method to improve solutions obtained by the global optimization method alone. In the considered optimization problem, due to many local minima of the objective function, it is necessary to apply EA, but the efficiency of EA (measured as

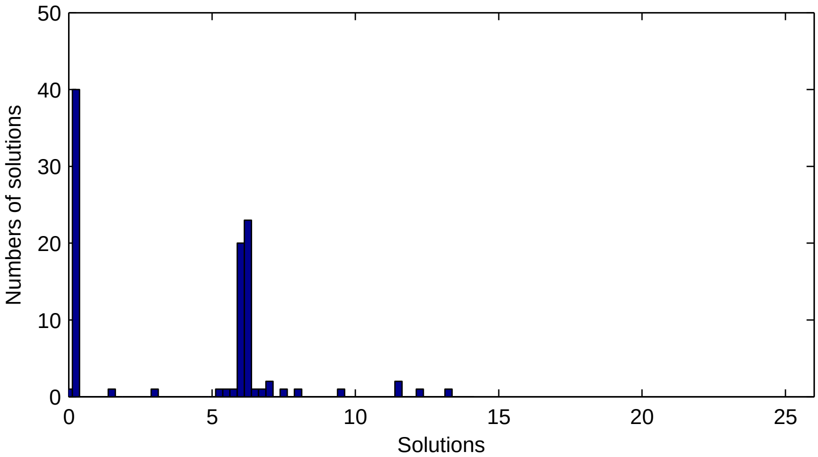

Figure 2. Histogram of generated solutions - Hybrid (500)

Table 3. Results for different iteration number of the EA

	EA			Hybrid method		
	solution		time	solution		time
number of EA iterations	mean	std	mean	mean	std	mean
5	26.62	5.19	0.24s.	8.30	4.69	1.21s.
10	21.98	5.32	0.48s.	8.16	5.32	1.54s.
20	20.12	4.56	0.96s.	7.12	3.59	1.98s.
50	17.55	3.73	2.41s.	6.62	3.67	3.35s.
100	16.18	3.43	4.81s.	6.60	3.67	5.75s.
200	12.94	3.37	9.95s.	5.18	3.93	10.94s.
500	10.51	2.99	24.00s.	3.99	3.38	24.86s.
1000	7.82	2.74	49.43s.	3.38	3.46	50.45s.

time, but assuming the same value of the mean solution) may be significantly improved using the presented hybrid method.

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