Multiobjective optimization of heat radiators using evolutionary algorithms

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Abstract. The paper is devoted to the multiobjective optimization of heat radiators using evolutionary algorithms. The proposed algorithm, based on the Pareto approach, is applied in optimization of heat radiators used to dissipate heat from electrical devices. The boundary-value problem for thermoelasticity is solved using the finite element method. Numerical examples are also included in the paper.

1 Introduction

In many real-world engineering problems several aims must be satisfied simultaneously in order to obtain an optimal solution. In the first phase of the design process the set of objectives is unclear and the designer has to define them as precisely as possible. Moreover, for the multiobjective optimization [2][7][8][9] the goals are usually in conflict with each other. For example, the volume of the radiator should be minimized while the total dissipated heat flux or maximal value of the equivalent stress should be maximized (or minimized also). The common approach in this sort of problems is to choose one objective (for example the volume of the structure) and incorporate the other objectives as constrains. This approach has been presented in previous paper [4][5][6][10], but it has the disadvantage of limiting the choices available to the designer, making the optimization process rather difficult.
The evolutionary algorithms using the Pareto approach are proposed as the optimization technique. The fitness function is calculated for each chromosome in each generation by solving a boundary value problem of thermoelasticity by means of the FEM [3][14]. The optimized radiators are modelled as structures subjected to mechanical and thermal boundary conditions. The interaction of stress and temperature fields is modelled by means of the theory of the thermoelasticity.

2 Multiobjective optimization

In the multiobjective optimization solution of the problem is represented by more than one objective function. In such problems all of the objective functions cannot be simultaneously improved, moreover they are usually in conflict with each other, so the term "optimize" means finding such a solution which would give the values of all objective functions acceptable to the

designer. Instead of one optimal solution the set of optimal solutions can be received. A multiobjective optimization problem can be expressed as follows:
Find the vector $\mathbf{x} = [x_1, x_2, \ldots x_n]^T$ which will satisfy the m inequality constrains:

$$g_i(\mathbf{x}) \geq 0 \qquad i = 1, 2, \ldots, m \tag{1}$$

and the p equality constrains

$$h_i(\mathbf{x}) = 0 \qquad i = 1, 2, \ldots, p \tag{2}$$

which minimizes the vector of k objective functions

$$f(\mathbf{x}) = \left[f_1(\mathbf{x}),\, f_2(\mathbf{x}),\, \ldots,\, f_k(\mathbf{x}) \right]^T \tag{3}$$

The set of objective functionals for multiobjective optimization of radiators can be defined as:

- the minimization the volume of the structure:

$$\min_{\mathbf{x}} V(\mathbf{x}) \tag{4}$$

- the minimization of the maximal value of the equivalent stress:

$$\min_{\mathbf{x}} \sigma_{eq}^{\max}(\mathbf{x}) \tag{5}$$

- the maximization of the total dissipated heat flux

$$\max_{\mathbf{x}} q(\mathbf{x}) \tag{6}$$

The multiobjective optimization is performed by using the Pareto concept. The Pareto-optimality is defined as a set $\mathcal{F}_P$, where every element f_P is a solution of the problem defined by Eqs. (1)-(3), for which no other solutions can be better with regard to all objective functions. In other words the solution is Pareto optimal if there exist no feasible vector which would decrease some criterion without causing a simultaneous increase of another criterion. In Figure 1 a bold line is used to marked the set of Pareto optimal solutions which is called the *Pareto front*.

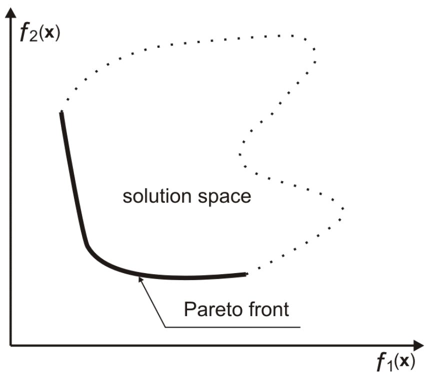

Figure 1. An example of the biobjective problem.

Considering two solution vectors $\mathbf{x}$ and $\mathbf{y}$ for a minimization problem, $\mathbf{x}$ is contained in the Pareto front if:

$$\begin{aligned} &\forall i \in 1, 2, \dots, k: \; f_i(\mathbf{x}) \le f_i(\mathbf{y}) \\ &and \\ &\exists j \in 1, 2, \dots, k: \; f_j(\mathbf{x}) < f_i(\mathbf{y}) \end{aligned} \tag{7}$$

The Pareto optimum always gives not a single solutions, but a set of solutions called non-dominated solutions or efficient solutions.

3 Multiobjective evolutionary algorithm

In order to solve the optimization problem the evolutionary algorithm [1][12] with the real-coded representation has been proposed. The solution of this problem is given by the best chromosome whose genes represent design parameters responsible for shape of heat radiator. The flow chart of the multiobjective evolutionary algorithm is shown in Figure 2.

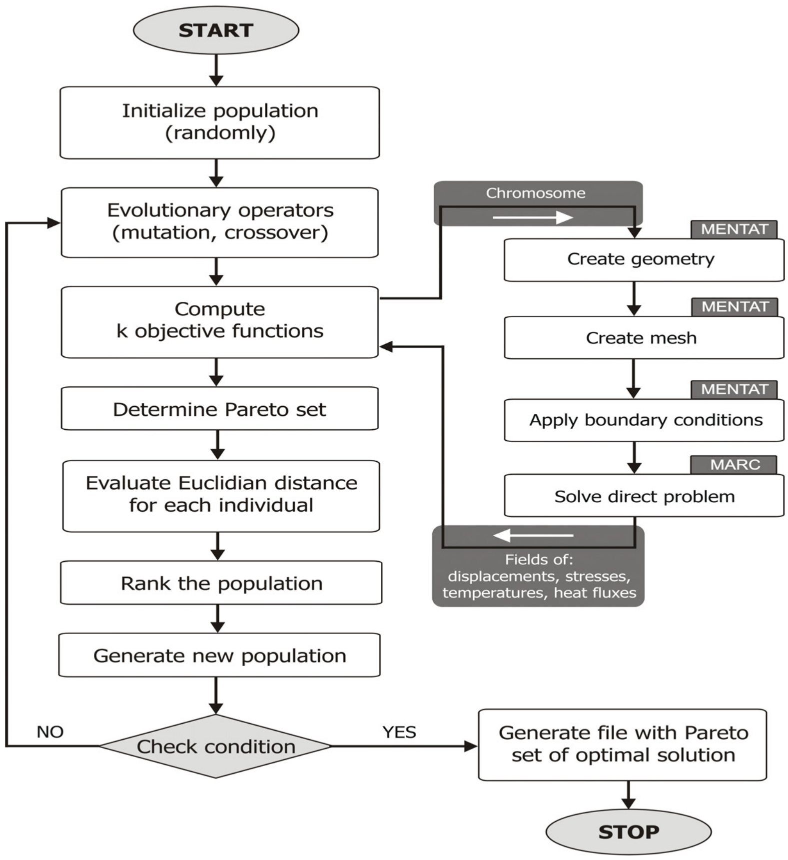

Figure 2. The flow chart of the multiobjective evolutionary algorithm.

The proposed evolutionary algorithm starts with a population of chromosomes randomly generated. Two kinds of the mutation are applied: an uniform mutation and a Gaussian mutation. The operator of the uniform mutation replaces a randomly chosen gene of the chromosome with the new random value. This value corresponds to the design parameter with its constrains. For the Gaussian mutation a new value of the gene is created with the use of Gaussian distribution. The operator of the simple crossover creates two new chromosomes from the two randomly selected chromosomes. Both chromosomes are cut in randomly position and merge together. In order to compute *k* objective functions the thermoelasticity problem is solved (more details about direct problem is described in Section 4).
The selection is performed on the base of a ranking method, information about Pareto optimal solutions and the similarity of solutions. This procedure is very similar to the method of selection proposed by Fonseca and Fleming [11].
The Pareto set is determined in the current population by using Eq. (7). The Euclidian distance between all chromosomes is defined as follows:

$$\mathrm{ED}(x_i;x_j)=\sqrt{\sum_{n=1}^{popsize}\left(x_i(n)-x_j(n)\right)^2} \tag{8}$$

The rank of each chromosome depends on the number of individuals by which is dominated and scaled value of the Euclidian distance. This scheme helps to conserve diversity in the population. The most similar chromosomes have less probability to survive.
The next iteration is performed if the stop condition is not fulfilled. The stop condition is expressed as the maximum number of iterations. The Pareto set in each generation is stored into file. On the basis of this files the collective Pareto set of optimal solution is generated.

4 Evaluation of the fitness function

The fitness function is computed with the use of the steady-state thermoelasticity. Elastic body occupied the domain Ω bounded by the boundary Γ is considered (Figure 3).

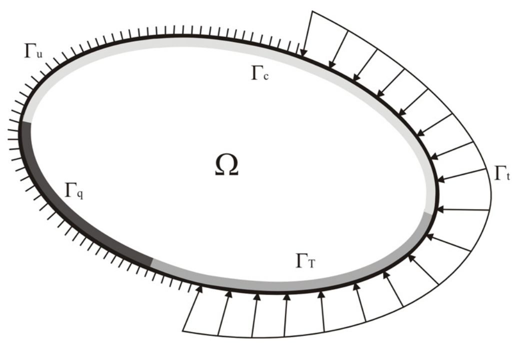

Figure 3. Elastic structure subjected to thermomechanical boundary conditions.

The governing equations of the linear elasticity and the steady-state heat conduction problem is expressed by the following equations:

$$G\,u_{i,jj} + \frac{G}{1-2\nu}u_{j,ji} + \frac{2G(1-\nu)}{1-2\nu}\alpha T_{,i} = 0 \tag{9}$$

$$\lambda T_{,ii} + Q = 0 \tag{10}$$

where G is a shear modulus and ν is a Poisson ratio, u_i is a field of displacements, α is heat conduction coefficient, λ is a thermal conductivity, T is a temperature and Q is an internal heat source.

The mechanical and thermal boundary conditions for the equations (9) and (10) take the form:

$$\begin{aligned} &\Gamma_t : t_i = \bar{t}_i \;\; ; \Gamma_u : u_i = \bar{u}_i \\ &\Gamma_T : T_i = \bar{T}_i \;\; ; \Gamma_q : q_i = \bar{q}_i \;\; ; \Gamma_c : q_i = \alpha(T_i - T^{\infty}) \end{aligned} \tag{11}$$

where $\bar{u}_i, \bar{t}_i, \bar{T}_i, \bar{q}_i, \alpha, T^{\infty}$ are known displacements, tractions, temperatures, heat fluxes, heat conduction coefficient and ambient temperature respectively.
Separate parts of the boundaries must fulfil the following relations:

$$\begin{aligned} &\Gamma = \Gamma_t \cup \Gamma_u = \Gamma_T \cup \Gamma_q \cup \Gamma_c \\ &\Gamma_t \cap \Gamma_u = \varnothing \\ &\Gamma_T \cap \Gamma_q \cap \Gamma_c = \varnothing \end{aligned} \tag{12}$$

In order to solve numerically thermoelasticity problem finite element method is proposed. After discretization taking into account boundary conditions following system of linear equations can be obtained:

$$\begin{aligned} &\mathbf{KU = F} \\ &\mathbf{ST = R} \end{aligned} \tag{13}$$

where **K** denotes stiffness matrix, **S** denotes conductivity matrix, **U**, **F**, **T**, **R** contain discretized values of the boundary displacements, forces, temperatures and heat fluxes.
This problem is solved by the FEM software – MENTAT/MARC [13]. The preprocessor MENTAT enables the production of the geometry, mesh, material properties and settings of the analysis. In order to evaluate the fitness function for each chromosome following four steps must be performed:

Step 1 *(generated using MENTAT)*
Create geometry and mesh on the base of the chromosome genes
Step 2 *(generated using MENTAT)*
Create the boundary conditions, material properties, settings of the analysis
Step 3 *(solved using MARC)*
Solves thermoelasticity problem
Step4
Calculate the fitness functions values on the base of the output MARC file

5 Numerical example

Consider a radiator whose cross-section is shown in Figure 4a. The structure is made of copper of following material properties: Young modulus E=110000Mpa, Poisson ratio $\nu = 0.35$, thermal expansion coefficient $\alpha = 16.5 \cdot 10^{-6}\ {}^{1}\!/_{K}$ and thermal conductivity $\lambda = 380\ {}^{W}\!/_{mK}$. Six design variables are assumed: the length of each fin (Z1-Z4), the width of the fins (the same for all fins – Z5) and thickness Z6. The geometry of the radiator is symmetric. The total width of the radiator is equal to 0.1m. Table 1 contains limitations of the design variables. Figure 4b shows thermo-mechanical boundary conditions. Force $P = 10N$ is applied on each fin. The temperature T^0, ambient temperature T^{ot} and the heat convection coefficient α is equal to $100°C, 25°C, 20\ {}^{W}\!/_{mK}$, respectively. The multiobjective problem is to determine the specific dimensions of the structure which minimizes the set of proposed functionals (4)-(6).

Table 1. The admissible values of the design parameters.

Design variable	Min value [m]	Max value [m]
Z1, Z2, Z3, Z4	0.01	0.05
Z5	0.0025	0.006
Z6	0.0025	0.008

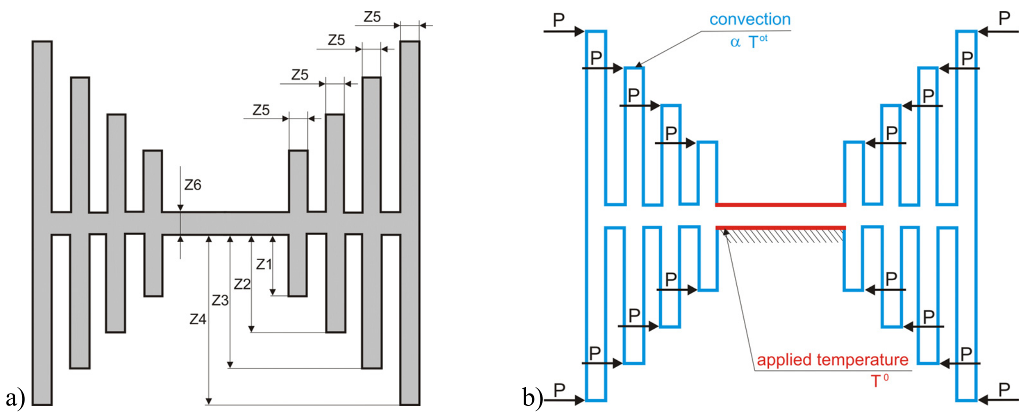

Figure 4. a) The design variables, b) The geometry and the boundary conditions

Several numerical experiments is performed. The set of Pareto optimal solutions with an example of the obtained shape for the minizmization both: the volume of the radiator (f_1) and the maximal value of the equivalent stresses (f_2) is presented in Figure 5a. Figure 5b contains the results for the maximization of the total dissipated heat flux and the minimization of the equivalent stresses (f_2) simultaneously. The set of Pareto solutions obtained for three proposed criterie (f_1 – volume, f_2 – equivalent stress, f_3 – heat flux), are presented in Figure 6.

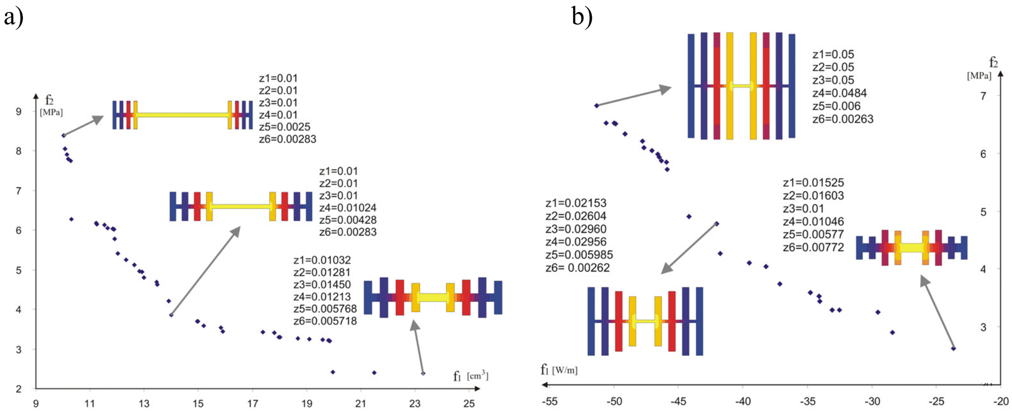

Figure 5. The set of Pareto optimal solution for the two criterion

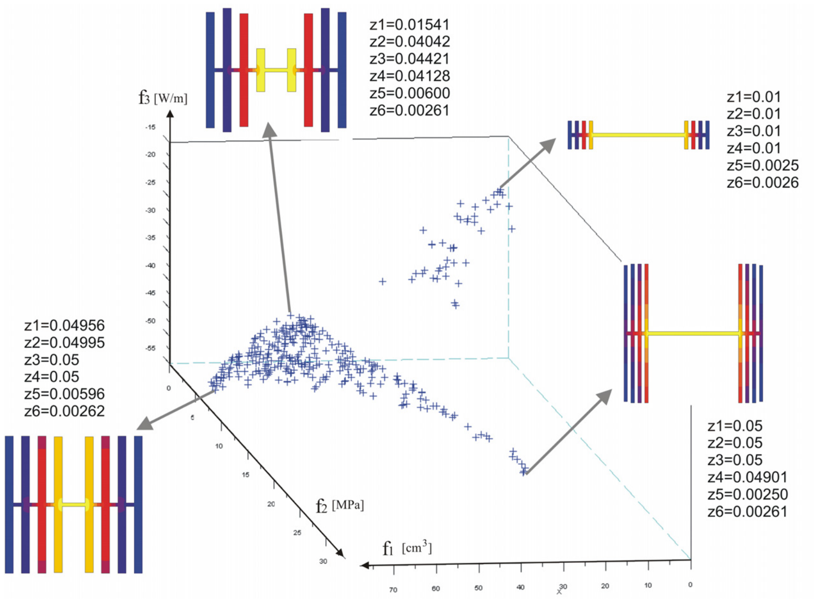

Figure 6. The set of Pareto optimal solution for the three criterion

6 Concluding remarks

The multiobjective shape optimization of heat radiators has been presented in the paper. The proposed multiobjective evolutionary algorithm gives the designer the set of optimal solutions based on more than one criterion. The choice of one objective and incorporate the other objectives as constrains requires performing optimization many times with different values of the constrains. Such approach makes the optimization process rather inadequate and difficult. Proposed approach is also more convenient than, for instance, to the “weighting method” in which fitness function is defined as a sum of objective functions and appropriate weights.

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