Evolutionary Identification of Fuzzy Material Constants in Laminates

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Abstract. The paper deals with the identification of material constants in fibre-reinforced laminates. The Evolutionary Algorithm is used as the global optimization method. The gradient method supported by the Artificial Neutral Network is used as the local optimization method. Material constants are presented in the form of fuzzy numbers to model their uncertainty. Two types of laminates are considered: simple and hybrid ones. Modal analysis methods are used to collect data necessary for the identification process. The Finite Element Method in fuzzy version is used to solve the direct problem for the laminates. Numerical examples are attached.

1 Introduction

The aim of the paper is to identify material constants in multilayered, fibre-reinforced laminates. Simple and hybrid laminates are considered. The material constants are treated as non-deterministic ones due to the manufacturing process. Uncertainties are introduced to reduce differences between real structures and their mathematical, ideal model. The inaccuracies can be modelled by using different granularity models, e.g. interval numbers, fuzzy numbers or in the stochastic way. The interval and fuzzy representation of the design variables is used in the present paper.

Composite is a material built by joining at least two materials on the macroscopic level. Composite usually consists of two permanently connected phases: i) the matrix, playing the role of the binder; ii) the reinforcement. Multi-layered laminates are the fibre-reinforced composites built of a specific number of stacked, permanently joined plies. Laminates made of polymer matrices with carbon, graphite, glass, boron or aramid fibers are the most typical ones. Plies in laminates are usually built of the same composite material but the angles of fibers usually placed directionally in the specific ply can vary. The main advantages of laminates, comparing with the isotropic materials are: i) the high strength-weight ratio; ii) the easiness of tailoring the material by manipulating: components material, stacking sequence, fibres orientation or layer thicknesses.

The cost of a laminate quickly increases as its strength rises. To ensure the high strength of the laminate reducing its cost, one composes the laminate of more than one material [1]. Typically, the exterior plies are made of the "better" and more expensive material, while the core of the laminate is made of the "worse" and cheaper one.

Composites are anisotropic and non-homogenous materials. Multi-layered and fibre-reinforced laminates can be usually treated as the 2-D orthotropic material with 4 independent elastic constants: axial Young's modulus E_1, transverse Young's modulus E_1, axial-transverse shear modulus G_{12} and axial-transverse Poisson ratio ν_{12} [8].

2 The Formulation of the Identification Task

The laminate elements are often manufactured individually or in short series. As a result, it is necessary to perform non-destructive tests to identify their elastic properties. To solve the identification problem it is required to measure the state fields' values.

The identification problems belong to inverse ones which are mathematically ill-posed [6]. The identification results strongly depend on the number of measurement data. To reduce the number of sensors, the dynamic properties of laminates can be taken into account and the modal analysis methods can be employed. The eigenfrequencies are used as the measurement data.

An eigenvalue problem for a laminate plate of length a, width b and thickness h in directions x, y and z, respectively, can be presented in the form [2]:

$$\rho h \omega^2 w = D_{11} w_{,xxxx} + D_{16} w_{,xxxy} + 2(D_{12} + 2D_{16}) w_{,xxyy} + 4D_{26} w_{,xyyy} + D_{22} w_{,yyyy} \quad (1)$$

where: w - deflection in the z direction; ω - eigenvalue vector; D_{ij} - bending stiffness; ρ - mass density.

The stiffness matrix for one lamina in in-axis orientation has the form:

$$\mathbf{Q} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\,\nu_{21}} & \frac{\nu_{21}\,E_1}{1-\nu_{12}\,\nu_{21}} & 0 \\ \frac{\nu_{12}\,E_2}{1-\nu_{12}\,\nu_{21}} & \frac{E_2}{1-\nu_{12}\,\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \quad (2)$$

where:

$$\nu_{21} = \nu_{12} \frac{E_2}{E_1} \quad (3)$$

The relation between matrices $\mathbf{Q}$ and $\mathbf{D}$ is as follows:

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N_p} (Q_{ij})_k (z_k^3 - z_{k-1}^3) \quad (4)$$

Each of elastic constants has the form of the interval or the fuzzy number. As a result, the elements of the $\mathbf{D}$ matrix also have the form of the interval or the fuzzy numbers.

The identification problem is defined as the minimization of the objective function with respect to the vector of the design variables $\mathbf{x}$:

$$\min : J(\mathbf{x}) = \min \left[\sum_{i=1}^{n} (q_i - \hat{q}_i)^2 \right] \quad (5)$$

where: $\mathbf{x} = (x_i)$ - the design parameters; $\hat{q}_i$ - measured values of a state fields; q_i - values of the same state fields for considered solution.

The vector of the design parameters consists of identified elastic constants. The sensor points are situated on the surface of the body. The fuzzy boundary-value problem is solved by using the fuzzy finite element method (FFEM). In the fuzzy representation the objective function is modified to the fuzzy form. In the case of the identification problem the edges of the intervals:

$$J_C(\mathbf{x}) \in \left[\underline{J}_C(\mathbf{x}), \overline{J}_C(\mathbf{x})\right] \tag{6}$$

are calculated as follows:

$$\begin{aligned} \underline{J}_C(x_j) &= \sum_i \min\left(\left|\underline{q}_i - \underline{\hat{q}}_i\right|, \left|\overline{q}_i - \overline{\hat{q}}_i\right|\right) \\ \overline{J}_C(x_j) &= \sum_i \max\left(\left|\underline{q}_i - \underline{\hat{q}}_i\right|, \left|\overline{q}_i - \overline{\hat{q}}_i\right|\right) \end{aligned} \tag{7}$$

The central values of the intervals are calculated as the mean values of both edges for all α-cuts. The vector of the design variables $\mathbf{x}$ has the following forms:

- for a simple laminate:

$$\mathbf{x} = (E_1, E_2, G_{12}, \nu_{12}) \tag{8}$$

- for a hybrid laminate:

$$\mathbf{x} = \left(E_1^1, E_2^1, G_{12}^1, \nu_{12}^1, \rho^1, E_1^2, E_2^2, G_{12}^2, \nu_{12}^2, \rho^2\right) \tag{9}$$

In the case of the hybrid laminate the densities of particular materials also have to be identified. The superscripts in Equation (9) denote the material number.

To solve the identification task the optimization methods are used. To solve the global optimization task and avoid problems with calculation of an objective function gradient the Evolutionary Algorithm is used [3]. Evolutionary identification method was successfully tested for the non-fuzzy (real-coding) laminates' elastic constants identification problems [4].

3 The Two-Stage Fuzzy Strategy

The main idea of the Two-Stage Fuzzy Strategy (TSFS) is to couple the advantages of: gradient optimization methods, evolutionary optimization and artificial neural networks. In the proposed method the fuzzy EA is used in the beginning to generate a cluster (population) of fuzzy points. After this stage it is assumed, that the population of points is situated in the contiguity of the optimum which is probably the global one. The moment of the stopping of the EA is crucial and is discussed widely in [11].

In the next step the local optimization method (LOM) is employed to finish computations. In this stage a few the best points from the cluster are selected as a cloud of points necessary in the second stage in which the local optimization method is used. The sensitivity of the fitness function is approximated by means of the ANN.

3.1 The Fuzzy Evolutionary Algorithm

In the present paper the fuzzy version of the Evolutionary Algorithm (FEA) is used. In the FEA the data representation, the evolutionary operators as well as the selection procedure are the fuzzy ones [7]. Each chromosome represents one fuzzy solution. After evaluation of the solution a fuzzy fitness function value of the chromosome is obtained.

The main difference between the Fuzzy Evolutionary Algorithm and the typical, real-coding Evolutionary Algorithm is that the FEA works on fuzzy chromosomes consisting of fuzzy genes [11].

The j-th fuzzy chromosome has the form:

$$ch^j(\mathbf{x}) = \left[x_1^j, x_2^j, ..., x_i^j, ..., x_N^j\right] \tag{10}$$

where: N - the number of genes in chromosome.

Each gene represents one fuzzy number and each chromosome is a potential fuzzy solution of the problem. Fuzzy numbers are subsets of the fuzzy sets, being convex and normalized fuzzy sets with continuous membership functions [9].

As the standard representation of the fuzzy number can be inconvenient from the fuzzy number arithmetic point of view, it is suitable to represent the fuzzy number x as a set of the interval values $[\underline{x}; \overline{x}]$ lying on the adequate α-cuts, as shown in Figure 1a.

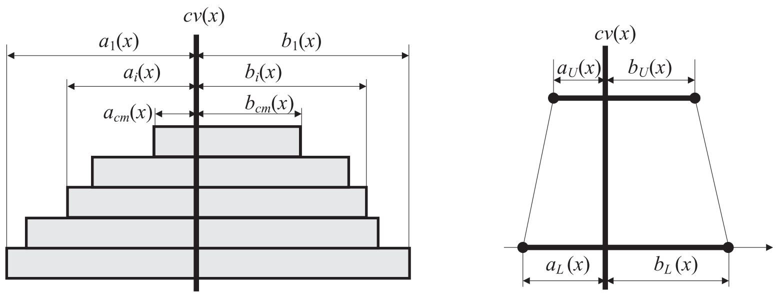


Figure 1. a) The fuzzy gene structure in the FEA; b) trapezoidal, asymmetric fuzzy gene.

In this attitude it is necessary to apply the interval arithmetic operators for each α-cut. Each gene in the chromosome is represented as a vector of the real values in one of two forms:

- interval representation:

$$x_i^j = \left[\underline{x}_i^j; \overline{x}_i^j\right] \tag{11}$$

- fuzzy representation:

$$x_i^j = \left[a_1(x_i^j), ..., a_{cm}(x_i^j), cv(x_i^j), b_{cm}(x_i^j), ..., b_1(x_i^j)\right] \tag{12}$$

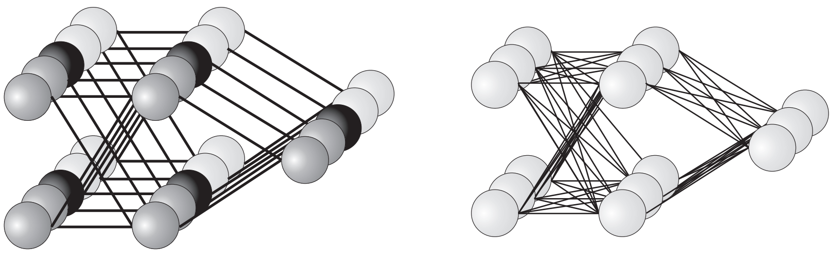
where: $cv(x_i^j)$ - the central value of a fuzzy number; $a_k(x_i^j), b_k(x_i^j)$ - distances between the central value and the left and right boundaries of the interval on k-th α-cut, respectively; cm - the number of α-cuts.

In the present paper each gene can be represented as: i) an interval number, or ii) an asymmetric, trapezoidal fuzzy number - each gene consists of 5 values (Figure 1b).

3.2 The Local Optimization Method

The proposed local optimization method (LOM) is a combination of the steepest descent method and the ANN networks. The number of neurons in the input layer is equal to the number of design variables of the fitness function. The output layer consists of one neuron the output value of which plays role of the fitness function. The number of neurons in hidden layers depends on the complexity of the fitness function.

The special multilevel ANN is used as the approximation tool of the fuzzy boundary-value problem. Each level of the ANN corresponds with selected parameter of the fuzzy number (Figure 2a).



a) b)

Figure 2. The scheme of the multi-level neural networks: a) with no level interconnections; b) with full level interconnections.

The number of the levels of multilevel neural network depends on the number of α-cuts. All levels can be interconnected (Figure 2b).

In the first stage of the LOM a cloud of fuzzy points is generated in the function domain. The optimization process is performed by means of the ANN. After training the ANN, the optimization process is performed by means of the gradient method (steepest descent method). The ANN output value is used as the approximation of the fitness function value. The special kind of a fuzzy gradient is introduced. For all edges of the fuzzy values the real sensitivities are calculated. Each parameter of the fuzzy number can be modified on the basis of such information.

As the last step the termination condition is checked. If it is true, the point from the previous step becomes a result of the optimization process, otherwise the considered point is added to the training vector set and the next iteration is carried out. The proposed local optimization method was successfully tested for the non-fuzzy problems [10].

4 Numerical examples

4.1 Identification of the Simple Laminate

A rectangular simple laminate plate made of the glass-epoxy is considered(Figure 3a). Each ply has thickness h=0.002m. The stacking sequence of the laminate is: (0/45/90/-45/0/90/0/90)s. The plate FEM model consists on 200 4-node plane finite elements. The identified elastic constant values are: E_1=3.86e10MPa, E_2=8.27e9MPa, G_{12}=4.14e9MPa, ν_{12}=0.26. The first 10 eigenfrequencies of the plate are taken into account in the identification procedure.

Each chromosome $ch^j(\mathbf{x})$ in population consists of 4 genes. Each gene x_i is a fuzzy number represented by 2 α-cuts (trapezoidal, asymmetrical fuzzy number) and described by 5 (non-fuzzy) values. The parameters of the FEA are: the number of chromosomes: $pop\_size = 100$; the number of generations: $gen\_num = 400$; arithmetic crossover probability: $p_c = 0.2$; gaussian mutation probability: $p_m = 0.4$.

The number of iterations of the local method (second stage) is 1000.

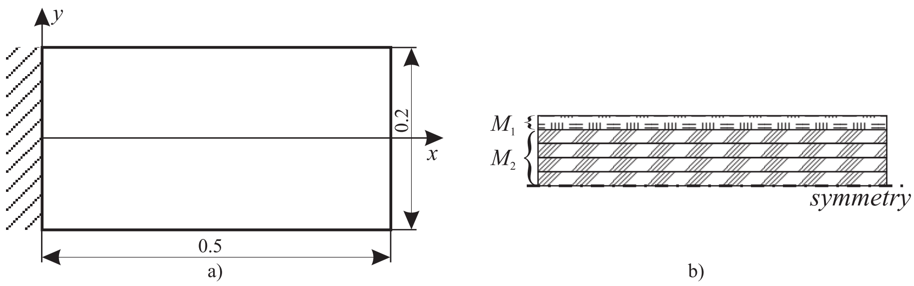


Figure 3. The laminate plate: a) shape and dimensions; b) hybrid laminate - materials location.

The variable ranges, actual values and results after the first stage (FEA) and after finishing computations using the local optimization method (LOM) are collected in Table 1 for the 1^{st} α-cut and in Table 2 for the 2^{nd} α-cut.

Table 1. The simple laminate - 1^{st} α-cut.

	$\underline{E}_1$ [MPa]	$\overline{E}_1$ [MPa]	$\underline{E}_2$ [MPa]	$\overline{E}_2$ [MPa]	$\underline{\nu}_{12}$	$\overline{\nu}_{12}$	$\underline{G}_{12}$ [MPa]	$\overline{G}_{12}$ [MPa]
Min	1.92E10	1.92E10	4.20E9	4.20E9	0.190	0.190	0.97E9	0.97E9
Max	5.08E10	5.08E10	10.80E9	10.80E9	0.410	0.410	8.03E9	8.03E9
Actual	**3.82E10**	**3.90E10**	**8.23E9**	**8.31E9**	**0.257**	**0.263**	**4.10E9**	**4.18E9**
After FEA	3.90E10	3.96E10	8.11E9	8.45E9	0.268	0.277	4.17E9	4.22E9
After LOM	3.82E10	3.90E10	8.23E9	8.31E9	0.257	0.263	4.10E9	4.18E9

Table 2. The simple laminate - 2^{nd} α-cut.

	$\underline{E}_1$ [MPa]	$\overline{E}_1$ [MPa]	$\underline{E}_2$ [MPa]	$\overline{E}_2$ [MPa]	$\underline{\nu}_{12}$	$\overline{\nu}_{12}$	$\underline{G}_{12}$ [MPa]	$\overline{G}_{12}$ [MPa]
Min	1.92E10	1.92E10	4.20E9	4.20E9	0.190	0.190	0.97E9	0.97E9
Max	5.08E10	5.08E10	10.80E9	10.80E9	0.410	0.410	8.03E9	8.03E9
Actual	**3.84E10**	**3.87E10**	**8.25E9**	**8.28E9**	**0.259**	**0.261**	**4.12E9**	**4.15E9**
After FEA	3.92E10	3.95E10	8.16E9	8.23E9	0.262	0.277	4.17E9	4.20E9
After LOM	3.84E10	3.87E10	8.25E9	8.28E9	0.259	0.261	4.12E9	4.15E9

4.2 Identification of the Hybrid Laminate

A rectangular hybrid laminate plate of dimensions presented in Figure 3a) is considered. Each ply has thickness h=0.002m. The stacking sequence of the laminate is: (0/15/-15/45/-45)s. The plate FEM model consists of 200 4-node plane finite elements. The first 10 eigenfrequencies of the plate are considered.

The external plies of the laminate are made of material M_1, the core plies are made of the material M_2 (Figure 3b). The material properties are collected in Table 3.

Table 3. The hybrid laminate - material parameters.

Material	E_1 [GPa]	E_2 [GPa]	ν_{12}	G_{12} [GPa]	ρ [kg/m^3]
M_1	181	10.3	0.28	7.17	1600
M_2	38.6	8.27	0.26	4.14	1800

Each of 10 genes x_i is a interval number represented by 2 (non-fuzzy) values. The parameters of the FEA are: the number of chromosomes: $pop\_size = 100$; the number of generations: $gen\_num = 1200$; arithmetic crossover probability: $p_c = 0.2$; gaussian mutation probability: $p_m = 0.4$.

The number of iterations of the local method (second stage) is 2500. The variable ranges, actual values and results after the first stage (FEA) and after finishing computations using the local optimization method (LOM) are collected in Table 4 for the material M_1 and in Table 5 for the material M_2.

Table 4. The hybrid laminate - material M_1.

	$\underline{E}_1$ [MPa]	$\overline{E}_1$ [MPa]	$\underline{E}_2$ [MPa]	$\overline{E}_2$ [MPa]	$\underline{\nu}_{12}$	$\overline{\nu}_{12}$
Min	1.52e10	1.52e10	4.20e9	4.20e9	0.190	0.190
Max	15.08e10	15.08e10	15.80e9	15.80e9	0.410	0.410
Actual	**3.82e10**	**3.90e10**	**8.23e9**	**8.31e9**	**0.257**	**0.263**
After FEA	3.74e10	4.12e10	8.21e9	8.22e9	0.273	0.278
After LOM	3.82e10	3.90e10	8.23e9	8.31e9	0.257	0.263
	$\underline{G}_{12}$ [MPa]	$\overline{G}_{12}$ [MPa]	$\underline{\rho}$ [kg/m^3]	$\overline{\rho}$ [kg/m^3]		
Min	0.97e9	0.97e9	0.97e3	0.97e3		
Max	8.03e9	8.03e9	3.03e3	3.03e3		
Actual	**4.10e9**	**4.18e9**	**1.80e3**	**1.81e3**		
After FEA	4.01e9	4.54e9	1.85e3	1.89e3		
After LOM	4.10e9	4.18e9	1.80e3	1.81e3		

Table 5. The hybrid laminate - material M_2.

	$\underline{E}_1$ [MPa]	$\overline{E}_1$ [MPa]	$\underline{E}_2$ [MPa]	$\overline{E}_2$ [MPa]	$\underline{\nu}_{12}$	$\overline{\nu}_{12}$
Min	1.52e10	1.52e10	4.20e9	4.20e9	0.190	0.190
Max	25.08e10	25.08e10	25.80e9	25.80e9	0.410	0.410
Actual	**18.00e10**	**18.20e10**	**10.00e9**	**10.04e9**	**0.277**	**0.283**
After FEA	17.20e10	18.26e10	9.87e9	10.36e9	0.273	0.284
After LOM	18.00e10	18.20e10	10.00e9	10.04e9	0.277	0.283
	$\underline{G}_{12}$ [MPa]	$\overline{G}_{12}$ [MPa]	$\underline{\rho}$ [kg/m^3]	$\overline{\rho}$ [kg/m^3]		
Min	0.97e9	0.97e9	0.97e3	0.97e3		
Max	8.03e9	8.03e9	3.03e3	3.03e3		
Actual	**7.10e9**	**7.18e9**	**1.60e3**	**1.65e3**		
After FEA	7.00e9	7.43e9	1.60e3	1.69e3		
After LOM	7.10e9	7.18e9	1.60e3	1.65e3		

5 Final Conclusions

The application of the Two-Stage Fuzzy Strategy for the simple and laminates‘ elastic constants identification has been presented. The strategy is a combination of the

Evolutionary Algorithms (AEs), the Artificial Neural Networks (ANNs) and the Local Optimization Methods (LOMs). The global optimization method in the form of the EA is used as the first step and then computations are finished by the local method. The ANN is employed to approximate the fitness function and the fitness function gradient.

The identified values and the fitness function are in the form of fuzzy or interval numbers. The strategy gives positive results for simple and hybrid laminates and can be applied for more complicated structures. If the number of data obtained from the eigenfrequencies is not sufficient it is also possible to use modal data from the frequency response of the structure. This attitude significantly reduces the number of sensor points comparing with static measurements (like strains or displacements).

For many real problems the computation of the fitness function by means of the finite element method is the most time-consuming part of calculations. It can be significantly reduced by means of the distributed computations [5].

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