Application of Hybrid Evolutionary Algorithm to Single Source Capacitated Warehouse Location Problem

Wojciech Jastrzębski – PhD student[1]

[1] Technical University of Lodz, Institute of Electronics, Lodz, Poland,
e-mail: wojastrz@p.lodz.pl

Abstract. This paper presents two-phase hybrid evolutionary algorithm (EA) to optimize Single Source Capacitated Warehouse Location Problem (SSCWLP); a well-known location-allocation problem employed for the telecommunication network design modeling. The first phase of the algorithm aims the search to satisfy the problem constraints and during the second phase actual optimization take place. To improve the performance of EA the algorithm is combined with other local search heuristics. Influence of EA hybridization as well as different selection schemes dealing with constraint handling are discussed. In addition, performance of co-evolutionary algorithm (co-EA) versus the EA with single population is compared across a set of example problems.

1 Introduction

The global telecommunications network is probably the largest and the most complicated structure devised by man. Correct planning and optimization of such network requires consideration of many complex factors like number, type, and distribution of the network elements over the geographical area as well as assignment between deployed network components. Moreover difficulty of producing cost-effective solutions increases with network size and complexity. So that, right design of telecommunications network infrastructure becomes very difficult and simultaneously very important task for the planner.

Modern telecommunication networks have a hierarchical structure and usually involve different design approaches. To tackle various network design issues number of location problems have been developed over the years, which can be then used within different phases of the network planning process. The aim of this paper is to present new hybrid evolutionary algorithm to the SSCWLP as a crucial location problem used during the telecommunication network design.

SSCWLP as most of the location problems relevant to the network optimization tasks can be classified as NP-complete. Exact and heuristic algorithms have been proposed to solve the problem [3, 9, 11]. Existing studies of the EA for the SSCWLP problem usually seeks for a specialized evolutionary operators and repair algorithms to deal with infeasible solutions. Approach proposed in this paper for simultaneous constraint handling and problem optimization, is the usage of two-phase hybrid evolutionary algorithm. Further solution improvement is achieved through co-evolutionary extension of the EA.

1.1 Basic Location Models in Telecommunication

Basic location models being relevant to the design of telecommunication networks can be roughly classified into number of different classes. These can be for instance uncapacitated and capacitated location models dealing with the nodes capacity constraints or dynamic models that consider the expansion of the telecommunication network over specific period of time [6].

Given this diversity of the network design models one can see that accurate planning and optimization of the today's modern networks involves application of various kinds of location problems. Capacitated location models engage special attention when dealing with telecommunication network design [8]. They are developed to decide about the deployment of concentrators and the assignment of end terminals without violating the capacity constraints of the concentrators. The capacity constraints may relate to the number of the terminals (inferior nodes) connected to the concentrator or might express terminal demands with subject to the consumed network resources (sort of commodity demands).

The paper presents the evolutionary approach for the solution of *Single Source Capacitated Warehouse Location Problem* (SSCWLP) with capacity constraints at node facilities.

1.2 Capacitated Warehouse Location Problem Statement

Warehouse Location Problems (WLP) belongs to the major class of location problem and a SSCWLP is a version of WLP involving single facility source for each customer with single commodity demands, limited warehouse capacity and single echelon [6, 8, 12].

Let's consider the problem of serving the given set of customers from the different warehouses. Assume that the set $I = \{1,.., m\}$ depicts of candidate sites we can locate warehouses and the set $J = \{1,.., n\}$ of customers needed supplier. Each warehouse $i \in I$ has his fixed cost f_i and constraint capacity s_i.. Each customer has a demand b_j, and c_{ij} is a cost of allocating all of the demand of customer j to warehouse i. The objective we want to reach is to find a subset of warehouses and to fully assign each customer to one of the chosen warehouse location in such a way that the sum of fixed costs of establishing warehouses and related transportation costs to supply the customer demand are minimized whilst the warehouse capacities are not exceeded. The problem is stated as follow:

$$\min\left(\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m} f_i y_i\right) \tag{1}$$

subject to:

$$\forall j \in J \qquad \sum_{i=1}^{m} x_{ij} = 1, \tag{2}$$

$$\forall i \in I \qquad \sum_{j=1}^{n} b_j x_{ij} \le s_i y_i \tag{3}$$

$$\forall i \in I, \forall i \in I \qquad x_{ij} \le y_j \tag{4}$$

$$\forall i \in I, \forall i \in I \qquad x_{ij} \in \{0,1\} \tag{5}$$

$$\forall i \in I, \qquad y_i \in \{0,1\} \tag{6}$$

where:

$$x_{ij} = \begin{cases} 1 & \text{if customer } j \text{ is supplied from warehouse } i \\ 0 & \text{otherwise} \end{cases} \qquad y_i = \begin{cases} 1 & \text{if warehouse is established at } i \\ 0 & \text{otherwise} \end{cases}$$

and:

f_i – cost of building a warehouse at i; c_{ji} – cost of supplying customer j from warehouse i

b_j – denotes demands of j –th customer; s_i – denotes warehouse capacity at location i

Constraint (2) ensures, that each client must be assigned to the single warehouse. Constraints (3) enforce the total demand of customers assigned to a facility not to exceed its maximum capacity. Constraint (4) ensures, that a client can only be assigned to an open warehouse. Two last constraints (5) and (6) represents the integrality requirements.

2 The Evolutionary Algorithm Approach – Two Phase Algorithm

This section deals with the implementation details of proposed two-phase hybrid evolutionary algorithm. The framework of the algorithm bases on the generic constrained optimization algorithm suggested in [13].

In the first phase of the algorithm the objective function is completely disregarded and the constrained warehouse location problem is treated as a constraint satisfaction problem. In the second phase the constraint satisfaction problem and optimization of the CWLP objective function are minimized simultaneous using bi-objective evolutionary algorithm.

Two selection schemes are incorporated into the second phase of the algorithm: *the preference schemes* and *non-dominated schemes.* For more information about constraint handling techniques reader is encourages to see papers [7,10,14].

The algorithm switches from phase one to the second phase after when at least single feasible solution is found. The pseudo-code of the proposed algorithm for solving SSCWLP is given by following:

```
Population initiation – customer based representation (see chapter 2.4)
  if (number of feasible solution =0)
  //PHASE1
        Goal: Minimize v(X) – constraints satisfaction problem
        Elite solution  => solution with least v(X)
        r(X) => rank-based individual fitness assignment based on violation v(X)
        Fitness function => ValFit1(X)=r(x)
        Tournament selection
        Apply genetic operators on population
  else
        //PHASE2
        f(X) – SSCWLP objective function
        Goal: Minimize (f(X),v(X)) – bi-criteria optimization
        Elite solution => feasible solution with least f(X)
        r(X) => rank based reproduction; two selection schemes
        d(X) => crowding distance operator; applied for the selection process (tournament)
        Fitness function => ValFit2 = r(X)
        Apply genetic operators on population
        Hybridization using local search heuristics
  end
generation=generation+1
end
```

Figure 1. Pseudocode of the proposed algorithm for WLP.

2.1 Constraints Satisfaction Problem

The goal of this phase is to find feasible solution from a random initialization.

Scalar objective function. The objective function calculation is done similarly to the penalty function when dealing with constrained optimization. From the formulation of SSCWLP we have *i* inequality constraints $g_i(X)<=0$ (i=1…m), where set $I = (1,...,m)$ defines warehouse locations, (compare constraint (3)). Constraint violation of individual X on the *i*-th constraint is calculated by:

$$c_i = \{\max(0, g_i(X))\} \qquad i = 1,...,m \tag{7}$$

where:

$$g_i(X) = \frac{\sum_{j=1}^{n} b_j \cdot x_{ij}}{s_i} \qquad \forall I \in \{1 \ldots m\} \tag{8}$$

characterizes degree of the constraint violation on the *i*-th facility.

The normalized scalar constraint violation function (9) takes values in the range (0,1).

$$v(X) = \frac{\sum_{i=1}^{m} \frac{c_i(X)}{c_{\max}(i)}}{m} \tag{9}$$

where:
c_{max} – denotes maximum violation of constraint *i*.

2.2 Bi-Objective Optimization – Constraint Optimization Problem

The goal of this phase is a global search of the optimum solution to the SSCWLP. In this phase the actual optimization take place. Objective functions (1) and (9) are minimized simultaneously using bi-objective EA.

Fitness function schemes. A major issue in solving the constraint optimization problem is the balance between selective pressure introduced by the selection schemes and to maintenance the population diversity. Thus two selection schemes are implemented to guide the search process, both based on the ranking selection: the *preference schemes* and *non-dominated schemes* [13].
The *preference scheme* is defined by the following:

- any feasible solution is better then any infeasible solution;
- among two feasible solutions *i* and *j,* assign greater probability of selection to the solution with the better objective function;

Regardless of the solution feasibility in case of individuals with the same objective value they receive the same rank.

The *preference scheme* is then compared with the *non-dominated scheme* where all solutions are ranked on base of the non-domination of their constraint violation and objective function values. All individuals are ranked on base of the Pareto-front they belong, according to [5].

When converging to the Pareto-optimal set of solutions it is desirable that the obtained set of solutions is spread equally over the Pareto-front. The diversity among non-dominated solutions is introduced by using the crowding comparison procedure [4]. The crowding distance estimates density of a neighborhood of a specific solution in the population and is used to guide tournament selection. Both algorithm phases use power rank-based fitness assignment adopted from [1].

Local search (LS) algorithm. A common approach to improve the performance of EA is to combine them with other search heuristics. They examine the effects of little local changes and accept only moves, which lead towards better solutions. The true advantage of the local search method is that this method does not need to be precise. Since only few steps of a local search algorithm may accelerate the convergence to the optimum [2].

Number of local search strategies can be devised for the SSCWLP. Examples of possible search methods are:

- change the customer assignment to another randomly chosen open site; accept all changes that improve solution simultaneously passing over all capacity constraints;
- for each customer change randomly site assignment; accept change only when beneficial and do not break capacity constraints;
- among open sites for each customer find the most beneficial feasible solution.

2.3 Co-Evolutionary Approach

In addition, to extend constrain handling scheme co-evolutionary approach is proposed alternatively to already described evolutionary algorithm. With this modification for the second phase instead of single population the algorithm processes number of subpopulation. This means that the global genetic operators will influence on each sub-population locally i.e. subpopulations will evolve independently exchanging the genetic information sporadically between each other [1]. Co-evaluating sub-populations use for phase 2 algorithm as proposed on Figure 1 and each employ unique local search operator (examples of LS strategies are described in previous subchapter). Since proposed local search methods differ is a way they treat SSCWLP constraints suggested modification of co-evolution shall increase diversity over the whole set of solutions improving balance between exploration and exploitation.

2.4 Evolutionary Algorithm Parameters

Described two-phase constrain handling evolutionary algorithm is applied to 12 test data derived from scientific database of OR Library and belong to standard test case scenarios for problems of WLP class.

For the SSCWLP encoding customer-based representation is used where each chromosome is a n-dimensional vector of integers in the set $\{1,m\}$: the integer value of the j-th position indicates the warehouse where the customer j is assigned. Used encoding always guarantee satisfaction of assignment constraints (2) and (4), see subchapter 1.2. Given representation allows manipulation of the genome using standard genetic operators such as uniform crossover and uniform mutation. The crossover probability was setup to $p_c=0.7$ and mutation ratio $p_m=0.01$.

The algorithm was realized in MATLAB environment. In the conducted experiments the fixed number of 240 individuals for EA and three subpopulations with 80 individuals each for co-evolutionary approach was chosen. For each phase elite number is setup to 1. There were no attempts to optimize genetic parameters i.e. crossover probability p_c, elite number and population size.

3 Results

To test proposed algorithm number of experiments was conducted using set of well-known test data from OR Beasley Library.

At first two proposed fitness assignment schemes are discussed. To evaluate both schemes Figure 2 plots the minimum, maximum, and mean fitness function values received for one of the SSCWLP problem versus generations. As observed from Figure 2 plot (a), higher spread between individuals is experienced for *non-dominated scheme*. Simultaneously when comparing the convergence to the global optimum across the set of test problems the *non-dominated scheme* slightly outperforms the *preference scheme.* This could be caused by the specificity of SSCWLP problem, where we should expect higher number of disconnected feasible solution. Therefore giving higher selection probability to infeasible individuals we help algorithm to explore more.

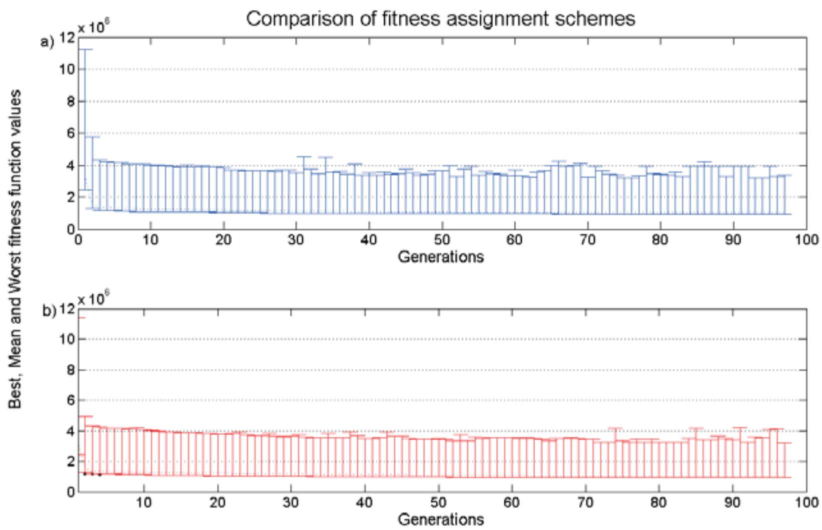

Figure 2. Best, worst and mean fitness function values versus generations using (a) nondominated scheme and (b) preference scheme.

Next figure presents the influence of incorporated local search strategies on the algorithm convergence to global optimum. Well-known approaches of LS method applications i.e. Boldwin and Lamarkian evolutions are compared against algorithm without LS method incorporated. As can be clearly seen the hybridization of EA speed up considerably the convergence to the global optimum increasing simultaneously the algorithm performance. Besides Lamarkian approach enables to explore the search space more efficiently comparing to Baldwin algorithm. For further tests local search operator is applied separately to each individual after genetic operators using Lamarkian approach.

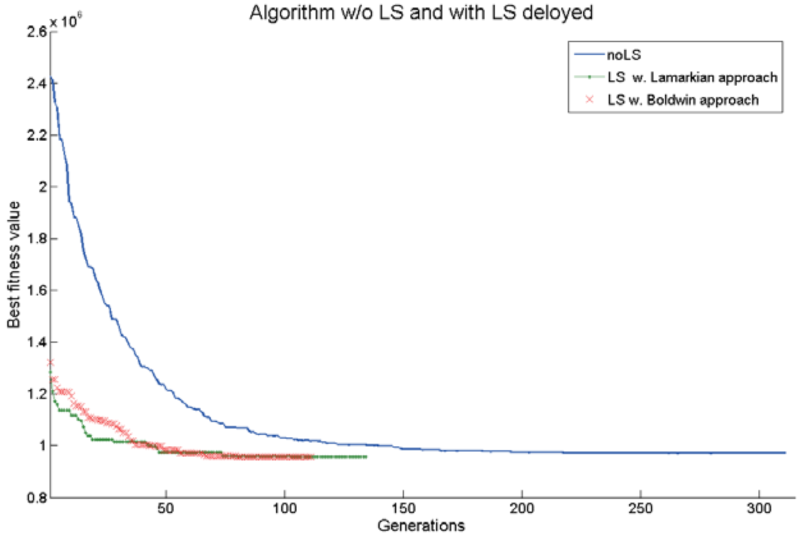

Figure 3. Best individual across generations for algorithm w/o local search with local search using Lamarkian and Boldwin approaches.

In the Table 1 the results acquired by two-phase hybrid EA on test data problems are presented. For every problem the algorithm was run 10 times limiting number of iterations to 500. The first column in the table identifies the test problem and their size (no of customers, no of sites) and the next present known optimum value. Further columns collect the performance indicators for 10 activations of algorithm. These are: the minimal obtained solution and number of solutions that reach global optimum, limit of 2% convergence and limit of 4 % convergence to the global optimum. In addition the performance of co-evolutionary algorithm for the same sort of test data is incorporated into table for comparison with traditional EA.

Table 1. The algorithm comparison: evolutionary algorithms vs. co-evolutionary approach.

Problem (Size)	Optimum value	10 runs of EA				10 runs of Co-EA			
		Best	Opt.	<2%	<4%	Best	Opt.	<2%	<4%
(25,50)									
Cap91	796648	**796648**	7	3	0	**796648**	5	5	0
Cap92	855733	858110	0	10	0	858110	0	10	0
Cap93	896617	905170	0	10	0	900760	0	9	1
Cap94	946051	957190	0	9	1	952430	0	8	2
(50,50)									
Cap121	793439	794810	0	10	0	794300	0	10	0
Cap122	852524	854900	0	10	0	854900	0	10	0
Cap123	895302	898870	0	10	0	899180	0	10	0
Cap124	946051	953040	0	6	4	951250	0	10	0

Test problems cap61 to cap63 for SSCWLP are omitted in the table because both EA and co-EA reach global optimum for each algorithm run. Analyzing the achievements of EA and co-EA no apparent discrepancy between the results can be noticed. Nevertheless it seems that co-EA manages better solution space. When compare more attentively it is seen that co-EA converges more closer to the known global optimum nearly for all test problems. Additionally for the largest test data for each run it reaches the borderline of 2% convergence to the optimum. This may be caused by improved population diversity naturally introduced by co-evaluating subpopulation as shown on Figure 4.

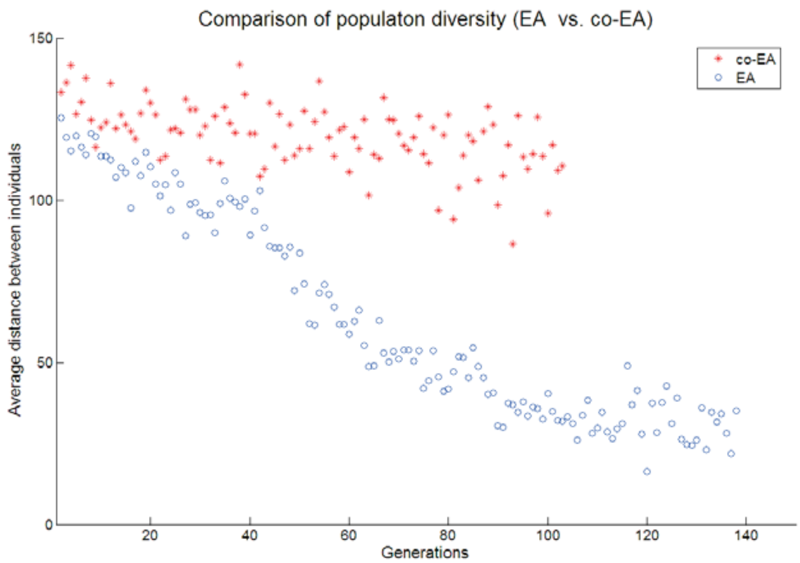

Figure 4. Average distance between individuals of EA and co-EA versus generations.

Figure 4 presents population diversity plotted versus generations using average distance measure between all individuals in the population. As showed using red markers '*' on the plot the proposed co-EA maintenance higher population diversity and as learned from the Table 1 is able simultaneously converges better to the optimum. Further increase of population diversity is expected from the usage of different local search strategies within each population. It should be also underlined that the processing time for both algorithms remains at the same level – since roughly speaking co-EA introduces only one additional step connected with migration between subpopulations.

4 Conclusion

Two-phase hybrid evolutionary algorithm was implemented to solve SSCWLP. To deal with constraints optimization on warehouse capacities methods based on the penalty function and multi-objective optimization were developed for a specific algorithm phases. The goal of the first phase is to find feasible solutions and within the second phase of the algorithm bi-objective optimization of SSCWLP problem starts.

The major issue in optimizing the constrained location problem is to maintain balance between exploitation and exploration. This is done by *non-dominated* rank-based fitness assignment and elitist introduction. Comparing to *preference scheme* it appears that EA with *non-dominated scheme* perform better. This specific fitness assignment scheme increases selection of individuals with some degree of infeasibility thus improving exploration of the search space. Also the notable improvement of the algorithm is achieved after applying local search operator with Lamarkian approach. As expected local search heuristic intensify the exploitation of the local minimum area attraction. Although application of additional local operators means longer time of processing nevertheless this can be recompensed by faster convergence. Further diversity preservation is obtained using co-evolutionary approach. Co-evoluting subpopulations implement various local search strategies within subpopulations. Along with Lamarkian evolution co-evolutionary algorithm enables to explore the search space more efficiently through smooth displacement of solutions into different parts of the search space. Co-EA converges more closely to the known global optimum nearly for all test problems

For tested problems proposed hybrid EA and co-EA extension was able to find optimum solutions or solutions close to the optimum.

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