# Adaptation Cost: what is the Repayment Time for the Evolutionary Saddle Crossings?

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**Abstract.** It is usual to asses alternative optimization search algorithms in terms of the number of trials necessary to approach the optimum. In case of adaptive algorithms implemented in the real systems such assessment seems inadequate, as the loss of quality due to the exploration of the vicinity of the identified optimum should be also taken into account. We try to evaluate this aspect of adaptive search in the case of a path dependent evolution with proportional selection and normally distributed mutations, which involves saddle crossing in a bimodal adaptive landscape.

## 1 Introduction

Presently, most of evolutionary algorithms and other algorithms of parallel search reside on computers and are engaged in optimization of computer models. In such applications the only factor that counts is the processing time. It might be expected that, in the relatively close future, the adaptive algorithms of parallel search for the improved solutions will increasingly become implemented to run not the simulated but the real life experiments (agent systems, swarms of robots, etc.). Even today, the algorithms of this type operate widely as institutional procedures [1] (without actually realizing this fact.)

The real life experiments are often inseparable from the normal operation of the system controlled by an adaptive algorithm. Then also the quality of the experimental solutions becomes important, as the poor solutions influence the overall performance. In such situations the adaptive dilemma emerges. The system might work in the best recognized way, tolerating no deviance from the optimal set of parameters, but then its abilities to react to the changes and to identify still better optima are nil. Or the system might explore widely for better solutions, but then its average performance might become intolerably lower than the currently possible. The natural approach is to look for a good compromise. It is easier said than done, as the loses from the lowered performance are evident, while the gains from the possible improvements are obscure at best. Still, it might be interesting to investigate this problem, looking for the usable hints.

Our contribution deals with the problem of the evolutionary saddle crossing [2]. The adaptive landscape is formed by two hills separated by a distinct saddle. The lower peak is already identified and the population fluctuates around it, in result of proportional selection and normally distributed mutations. These fluctuations lead eventually to the crossing of the saddle and the increase of the average fitness. In the mean time the fitness of successive individuals is generally lower then the fitness of the clones residing at the local optimum would be. The difference between the fitness of an individual and that of the clone is interpreted as the individual's cost. The sum of such cost carried through all individuals in successive generations is interpreted as the cumulative cost of adaptation. Once the saddle is crossed, and the average fitness becomes higher than the fitness of the previously optimal clones, the cumulative cost of adaptation starts to be repaid. When it gets down to zero it might be said that the saddle-crossing venture has turned to be profitable.

It might be noted that our approach to the cost of adaptation differs from the one discussed in the biological theory of evolution. There the *cost of evolution* is regarded in terms of the total reproduction rate required by the evolutionary scenario and is composed of cost of continuity, cost of mutation, cost of segregation, cost of random loss, cost of substitution and a few other minor costs [3].

Our interest is with the impact of the model parameters (population size, mutation variance, search space dimension) on the effect of the saddle crossing repayment. The paper is organized as follows: the model of evolution is described in Section 2; adaptation costs are defined in Section 3; the data on numerical experiments are presented in Section 4; Section 5 presents and discusses simulation results; Section 6 concludes the paper.

## 2 Model of Evolution

The simple model of phenotypic, asexual evolution with no-overlapping generations is considered [2]. The population of $m$ individuals evolves in the unbounded $n$-dimensional search space. The type of every individual $\boldsymbol{x}$ is given by a vector of its real valued traits $\boldsymbol{x}=(x_1, x_2, \dots x_n)$, and the fitness value $q(\boldsymbol{x})$. The fitness function $q(.)$ generates the adaptive landscape. It is assumed that the landscape is multimodal and consists of adaptive hills separated by adaptive saddles. Reproduction process proceeds in two steps:

1. **Selection.** Parents for individuals from the new generation are selected, with probability proportional to their fitness – proportional selection.
2. **Mutation.** Descendants inherit slightly modified parental traits. Each trait is mutated by adding a value of the normally distributed variable, with mean 0 and variance $\sigma^2$. The value of $\sigma$ is small with regard to the linear dimension of the adaptive landscape.

This simple model captures the essentials of the Darwinian evolution. The process is path-dependent. In the first phase the initially generated population concentrates quickly into a cluster of types, with a radius of about $\sigma$ [4]. Next this cluster moves systematically toward the more elevated areas of the adaptive landscape. This mode of evolution ends when the population comes close to some prominent adaptive peak (local optimum). Then the population enters the stage of selection-mutation equilibrium, and fluctuates haphazardly around this peak, with the average fitness distinctly lower than the locally optimal. The population might escape from such evolutionary trap, if there is a saddle that leads to some more attractive peak. The saddle crossing has a character of the random drift. Once it is done, and the more fit types behind the saddle are found (more fit than the average fitness in the selection-mutation equilibrium), another episode of the systematic evolution follows. In landscapes of the distinct adaptive hills such episodes of evolutionary progress are much shorter then the time spend in evolutionary traps. In effect the process described lead to evolution characterized by punctuated equilibria [5].

Our experience with the model described indicates that it is very efficient in saddle crossing, even when these saddles are tens of $\sigma$ wide, especially if the populations are very small [2,6]. This model is also very robust with regard to fitness perturbations [7] and modifications of the shape of adaptive landscapes [8]. Its saddle crossing efficiency gets significantly lower when the usual local optima zooming measures are introduced, such as recombination or intensification of selection [9].

## 3 Measures of the adaptation cost and the repayment time

The cost of adaptation of the *i*-th individual from the *j*-th generation is defined as a difference between the fitness $q^*$ of locally optimal type $\mathbf{x}^*$ and the fitness of the individual's type $\mathbf{x}_i^j$. The adaptation cost of the *j*-th generation $C^j$ is given by the formula:

$$C^j = \sum_{i=1}^{m} \left( q^* - q(\mathbf{x}_i^j) \right). \qquad (1)$$

The *accumulated cost* of adaptation after *k* generations is given by:

$$C_c^k = \sum_{j=1}^{k} C^j . \qquad (2)$$

The accumulated cost grows until the saddle is crossed[1]. Since then $C^j$ might be negative and the cumulative cost might diminish. The *repayment time* $T_R$ is defined by the number of generations necessary to make the cumulative cost negative:

$$T_R = \min_k \{ k : C_c^k < 0 \} .$$

Except of the repayment time it is convenient to consider also the *repayment effort*, measured by the number of trials necessary to reach the repayment time: $E_R = m \cdot T_R$.

## 4 Numerical experiments

The adaptation cost of crossing a saddle between optima of a fitness function was tested on the *n*-dimensional fitness function represented by a sum of two Gaussian functions (Figure 1):

$$q(\mathbf{x}) = \exp(-5\sum_{k=1}^{n} x_k^2) + 2 \cdot \exp(-5(1-x_1)^2 + \sum_{k=2}^{n} x_k^2) . \qquad (3)$$

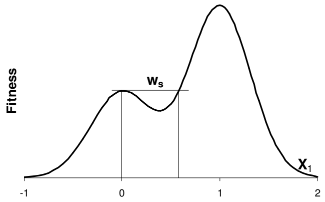

**Figure 1**. The cross-section of the n-dimensional adaptive landscape (3) along its $w_s$ wide saddle.

The main interest was in the influence of population size on expected repayment times in case of saddles of the various width and dimension. Simulations started with the initial populations of *m* identical individuals located at the local optimum. Runs were terminated after $T_R$ generations; i.e. when the cumulative cost (2) has turned negative. As the saddle crossings involved in the process are highly stochastic, the results from 2500 independent runs were averaged, to obtain reliable estimates of the expected values of $T_R$ (variance of the order of 2%).

The total number such of experiments was 84, as the series of simulations were repeated for the problem dimension *n* equal 2, 4 and 8; population size *m* equal 4, 8, 16 and 32, and the

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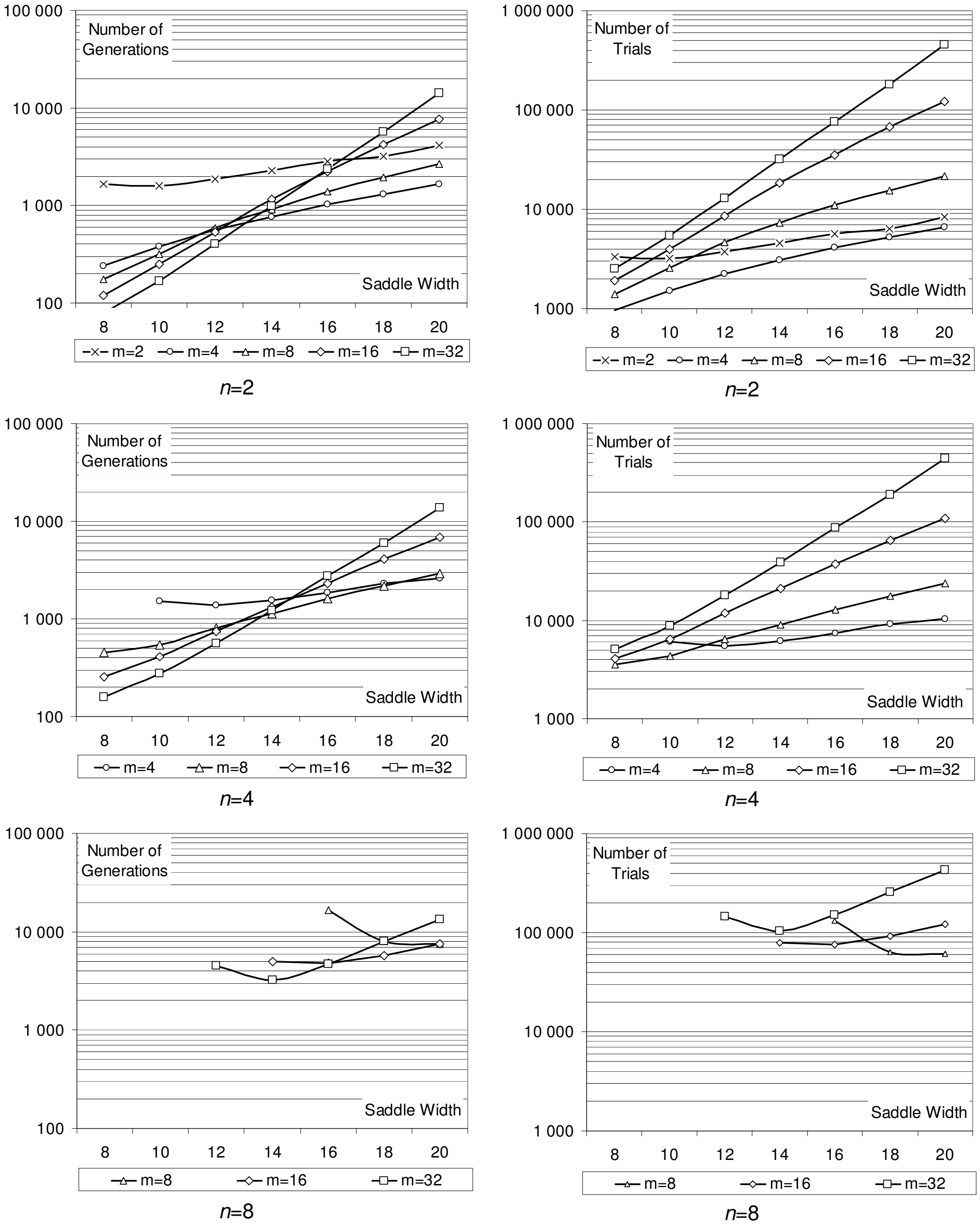
[1] Note, that value of $q^*$ responds to the local optimum and is fixed during calculation of the accumulated cost, even then the saddle is crossed.

standard deviation of modification σ set at $w_s/8$, $w_s/10 - w_s/20$. Later on, crossing of the saddle $w_s$ with σ= $w_s/k$ is referred as crossing of the saddle that is $k$σ wide.



*n*=2 *n*=2

*n*=4 *n*=4

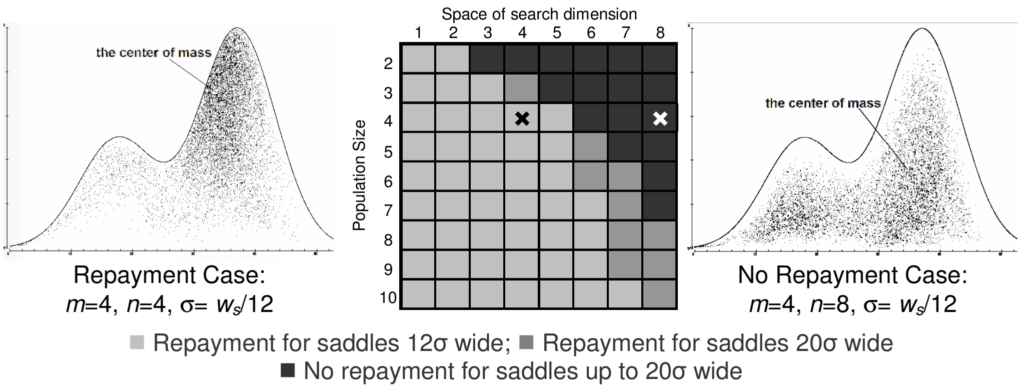
*n*=8 *n*=8

**Figure 2.** The repayment time $T_R$ (left column) and the repayment effort $E_R$ (right column) expected in adaptations involving crossing of $n$-dimensional saddles by populations of $m$ individuals. Saddle width are given in the units of σ.

## 5 Simulation results

The main body of results is presented in Figure 2. The expected repayment times $T_R$ and repayment efforts $E_R$ are given in the right and left column respectively. It might be noted that in case of wide saddles the repayment times are shorter for smaller population. For narrow saddles, the reverse might be true, especially if the dimension of the space of search is low. For saddles of about (12–14)·σ wide, the repayment time is almost independent of the population size. Situation is less ambiguous when the repayment efforts are considered. Small populations repay adaptive costs with fewer trials than large populations. This advantage increase significantly with the problem difficulty represented by the relative saddle width $w_s/\sigma$ and is moderately influenced by the dimension of the search space $n$.

Another effect, which might be observed in Figure 2, is that with $n$ growing the very small populations might fail to repay the adaptive cost at all. E.g. for populations of two the cost is never repaid when $n > 2$. This is due to the fact that when mutation size is considerable with regard to the linear size of the adaptive landscape (σ not much less then $w_s$) the average fitness in the mutation-selection quasi-equilibrium might be significantly lower then that of the local optimum. In our example the fitness at the global optimum is twice as high as in local optimum. If the fitness in quasi-equilibrium is less then half of the optimal fitness, the saddle crossing brings the improved results that are still worse that the locally optimal clones. This point to some inadequacy of the criterion used. The issue is illustrated in Figure 3.



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| 2 | ░ | ░ | █ | █ | █ | █ | █ | █ |
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**Figure 3.** Central panel: the diagram showing the repayment combinations of $n$ and $m$ for saddles 12σ and 20σ wide. Left and right panel: exemplary traces of evolution of a small population ($m$=4) mapped on the cross-section of the dimensional adaptive landscape ($n$=4, 8). The center of mass of current generation is depicted.

The Figure 4 allows some insight into the process of the adaptation cost repayment. Eight instances of simulations are presented. They are representative in the sense that the repayment times $T_R$ are close to the expected values of $T_R$ shown in the Figure 2. The top panels show the plots of the population average fitness (the values smoothed over the period of 11 generations). It might be observed that the average fitness in quasi-equilibria gets lower with the growth of $n$, and the repayment time grows accordingly. It might be also observed that the average fitness of larger populations tends to be greater than the fitness of smaller populations.

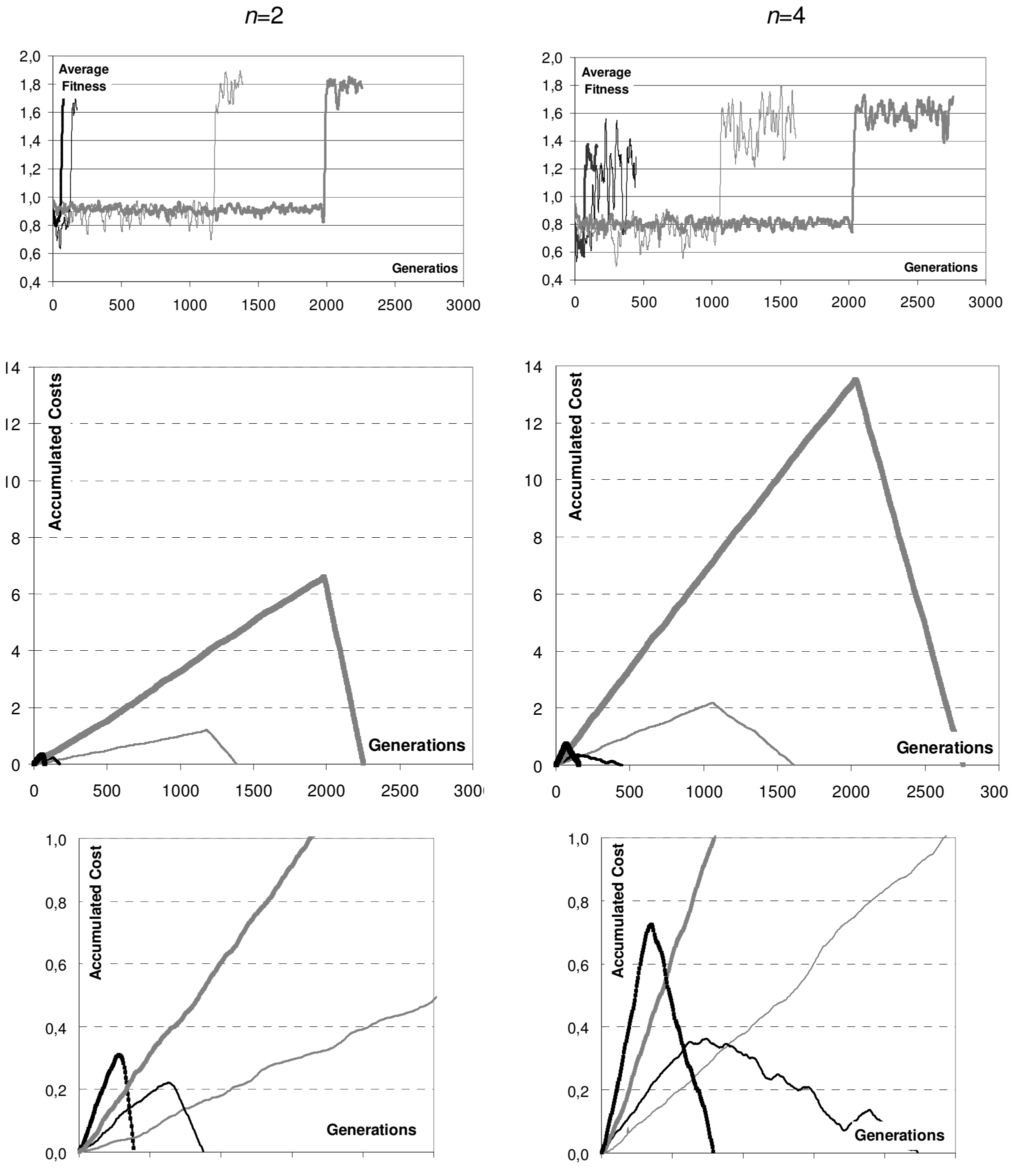

**Figure 4.** Typical instances of the saddle crossing related repayment in 2- and 4-dimensional search spaces. Saddles 8σ and 16σ wide are crossed by populations of 8 and 32 individuals, starting at the local optimum. Simulations terminated once the repayment is done. The top panel presents the averaged population fitness in successive generations. The middle panel presents the related curves of accumulated costs. The initial parts of these curves are shown in more detail in the bottom panel.

The middle panels and the bottom ones of the Figure 4 show the plots of the accumulated cost of adaptation. It might be observed that the costs are growing until the moment of saddle crossing and then start to get lower as the “losses” turn into “profits”. It might be observed that the accumulated costs are much lower for the small populations, even if the repayment times are longer. The cost effectiveness of small populations is especially striking in the case of wider saddles.

## 6 Conclusions

The experiments related allow for the following conclusion:

- Small populations incur lower cumulative costs of adaptation.
- In the cases of narrow saddles larger population might provide shorter times of repayment (in terms of generation numbers).
- In all cases smaller population allowed for the lower repayment effort (in terms of the number of trials).
- To shorten the time of repayment it is advisable to divide large population into several smaller ones and run evolutionary processes in parallel [10], until the saddle is crossed.

It might be noticed that in the case of large populations it might be profitable to delegate a small subpopulation to evolve under soft selection, and to incur hard selection (e.g. quality control) to keep the rest in the local optimum until the saddles is crossed. This might explain the winning strategy of the industrial revolution: production of optimal clones in factories and experimenting with diversity in laboratories.

There remains an open problem of the more sophisticated criteria of repayment. There are needed to deal with the cases when saddle crossings provide distinct improvement with regard to averaged fitness of the population fluctuating at the local optimum, but not enough improvement to make solutions better than the locally optimal clones.

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