Parallel evolutionary algorithms in shape optimization of heat exchangers under thermomechanical loading*

Adam Długosz[1], Tadeusz Burczyński[1,2], Wacław Kuś[1]

[1] Department for Strength of Materials and Computational Mechanics, Silesian University of Technology, Konarskiego 18a, 44-100 Gliwice

email: Adam.Dlugosz@polsl.pl, Tadeusz.Burczynski@polsl.pl, Waclaw.Kus@polsl.pl

[2] Institute of Computer Modelling, Cracow University of Technology, Warszawska 24, 31-455 Cracow

Abstract. The paper is devoted to the shape optimization of the heat exchangers under thermomechanical loading. To solve the problem the evolutionary algorithm is used as the optimization technique The fitness function is computed by means of the FEM commercial software - MSC MARC/MENTAT. In order to reduce the number of design parameters in evolutionary algorithms the shape of the structure is modelled by Bezier curves. Numerical examples for some shape optimization are included.

1 Introduction

The evolutionary algorithms have various applications in structural optimization. The main feature of this class of procedures is their randomness. The application of the evolutionary algorithms in optimization needs only information about the values of an objective (fitness) function. No sensitivity coefficients are required and the algorithms are able to find the global minimum in the presence of local minima. The main drawback of this techniques is their high computation cost. In order to speed up evolutionary optimization parallel evolutionary algorithms[10][7] is proposed instead of sequential evolutionary algorithms. The fitness function is calculated for each chromosome in each generation by solving a boundary value problem of thermoelasticity by means of the finite element method[2][14] (FEM). The optimized heat exchangers are modeled as structures subjected to mechanical and thermal boundary conditions[3][4][9]. Besides typical thermal boundary conditions radiation are taken into account. The geometry, mesh and boundary conditions are created on the basis of script language implemented in MENTAT. Another benefit of this approach is that MENTAT takes into account the shadowing effect in radiation.

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2 Formulation of the problem

The problem of the optimal shape of a heat radiator used to dissipate heat from electrical devices is considered. This problem is solved by the minimization of the following functionals:

- The minimum volume of the structure defined as:

$$\min_{X} V(X) \tag{1}$$

  with imposed constrains on the maximal value of temperature ($T - T^{ad} \leqslant 0$) and the maximal value of equivalent stress ($\sigma_{eq} - \sigma_{eq}^{ad} \leqslant 0$).
- The minimization of the maximal value of the equivalent stress defined as:

$$\min_{X} \sigma_{eq}^{\max}(X) \tag{2}$$

- The minimization of the maximal value of the temperature in the structure defined as:

$$\min_{X} T^{\max}(X) \tag{3}$$

  with imposed constrains on the maximal value of volume of the structure ($V - V^{ad} \leqslant 0$).

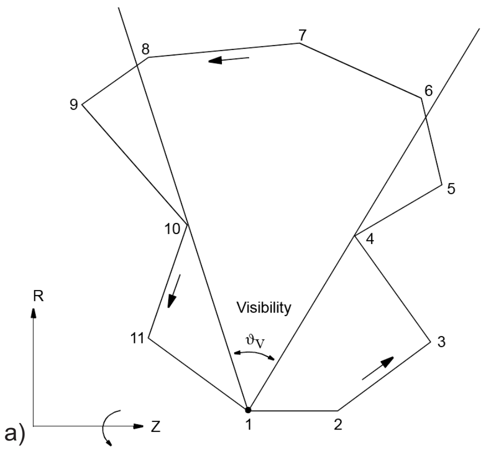
X is the vector of design parameters which is represented by a chromosome with the floating point representation. The heat radiator is modeled as a two dimensional (2D) plain stress problem. The fitness function is created by the method of penalty function taking into account: the volume of the structure, maximal value of the equivalent stress, maximal value of the temperature in the structure and imposed constrains.

3 Fitness function evaluation

The fitness function is computed by means of the theory of thermoelasticity. The commercial FEM[2][14] software - MSC MARC/MENTAT[8] is used to find the value of this function.

In the modeled structure mechanical as well as thermal boundary conditions are applied. Besides applied heat and convection, radiative boundary conditions are also taken into account. In many analyses, the radiative transfer[13][12] of heat between surfaces plays a significant role. To model this effect properly, it is necessary to compute the portion of one surface which is visible from the other surface known as the viewfactor. It is necessary to subdivide the radiative boundary in a heat transfer problem into one or more unconnected cavities. For each cavity, the system defines its outline in terms of an ordered sequence of nodes Figure 1a[8].

The preprocessor MENTAT enables the calculation of the viewfactor which is generally nontrivial. The internal script language implemented in MENTAT also makes the production of the geometry, mesh, material properties and settings of the analysis possible. Figure 1b shows the main steps of the evaluation of the fitness function for each chromosome.



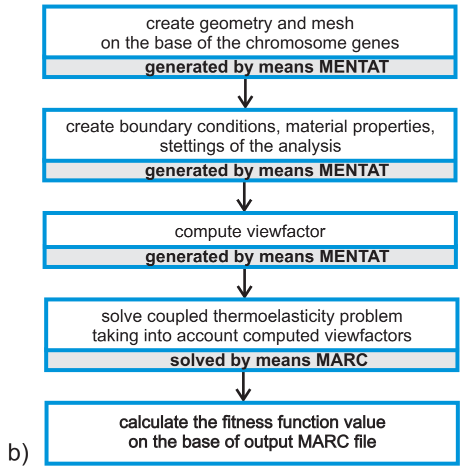


Figure 1. a) The radiating cavity, b) The algorithm of the calculation of the fitness function value

4 Parallel evolutionary algorithms

The sequential genetic algorithms [1][11] can be considered as the modified and generalized genetic algorithm in which chromosomes are coded by the floating point representation and various types of crossover and mutation operators are applied. The solution of this problem is given by the best chromosome whose genes represent design parameters responsible for the shape of the radiator. The evolutionary algorithm starts with a population of chromosomes randomly generated from the feasible solution domain. The design vector is represented by a chromosome X which consists of genes x_i, $i = 1, ...N$.

$$\mathrm{X} = [\mathrm{x}_1, \ldots \mathrm{x}_\mathrm{i}, \ldots \mathrm{x}_\mathrm{N}] \tag{4}$$

Genes can be considered as design variables. The following constrains are imposed on each gene:

$$\mathrm{x}_{\mathrm{iL}} \leqslant \mathrm{x}_\mathrm{i} \leqslant \mathrm{x}_{\mathrm{iR}},\ \mathrm{i} = 1{,}2, \ldots, \mathrm{N} \tag{5}$$

Two kinds of the mutation are applied: the uniform mutation and the Gaussian mutation. The operator of the uniform mutation replaces a randomly chosen gene of the chromosome with the new random value. This value corresponds to the design parameter with its con-strains. For the Gaussian mutation a new value of the gene is created with the use of Gaussian distribution. The probability of the mutation decides how many genes will be modified in each population. The operator of the simple crossover creates two new chromosomes from the two randomly selected chromosomes. Both chromosomes are cut in random position and coupled [1][11]. The ranking selection allows chromosomes with the great value of a fitness function to survive. The first step of the ranking selection is sorting all the chromosomes according to the value of the fitness function. Then on the basis of the position in the population the probability of surviving is attributed to every chromosome.

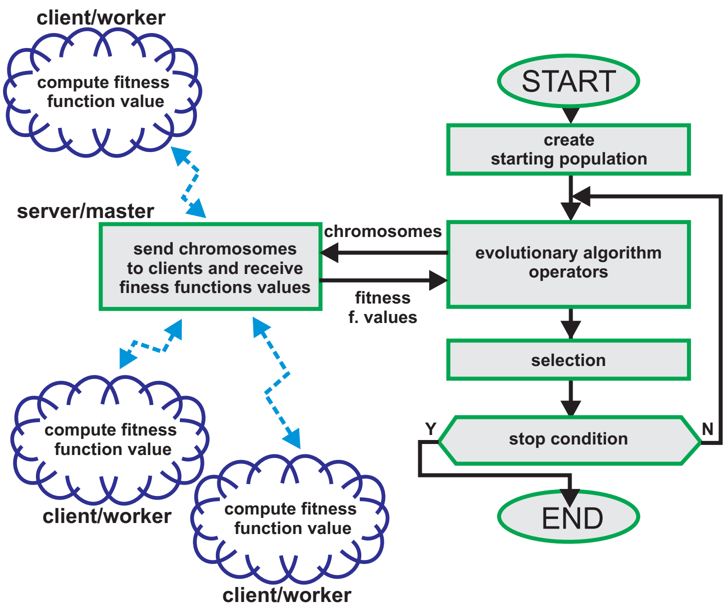


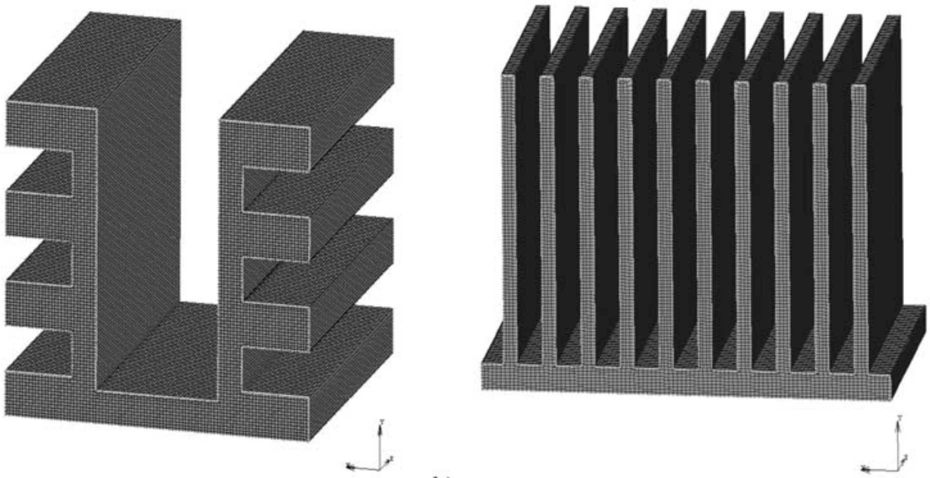
Figure 2. The flowchart of the parallel evolutionary algorithm

Sequential genetic and evolutionary algorithms are well known and applied in many areas of optimization. The main disadvantage of these algorithms is the long time needed for computing. The parallel evolutionary algorithms [10][7] perform evolutionary process in the same manner as their sequential counterparts. The difference is in fitness function evaluation. While for the sequential processes all members of the population are processed by the same CPU, in the case of parallel algorithms the values of the fitness function are calculated concurrently.

The approach used in this study was to allot the entire task of computing the fitness function corresponding to one chromosome to one processor unit. In this case the maximum (wall clock) computing time is shorter than its sequential counterpart nearly N times, where N denotes the number of involved CPUs. Small overhead comes from mutual communication between the units and evaluation of a new populations.

The flowchart of the parallel evolutionary algorithm[6][5] is shown in Figure 2. As already mentioned, the starting population is created randomly. The evolutionary operators change the values of the genes in the chromosomes and the fitness function value for each chromosome is computed.

The server/master transfers chromosomes to clients/slaves. The slaves compute the values of the fitness function and transmit them to the master. The generation of a new population is carried out by the server when the values of the fitness functions corresponding to each member of the old population are available. The creation of the new population is a random process. The probability for the fitter chromosomes to be included in the new population is higher. The process of generating new populations is terminated when the stop criterion is fulfilled. The latter condition was defined by defining the maximum number of iterations



a) b)

Figure 3. The two types of considered heat radiators

5 Numerical examples

The aim of the optimization is to find the optimal shape of the heat exchangers used to dissipate heat from the electrical devices shown in Figure 3. The invariable dimensions and the values of boundary conditions along Z axis are assumed. Due to above reasons a problem is modeled as two dimensional (2D). The radiators are made of copper whose material properties are shown in Table 1. Probability of simple crossover and Gaussian mutation is 0.5 and 1 respectively, whereas rank selection pressure is equal to 0.8.

Table 1. Material properties

Parameter	Value
Young modulus	$120000MPa$
Poisson ratio	0.3
Thermal expansion coefficient	$16.5E^{-6} 1/K$
Heat conductivity	$400W/mK$
Emissivity	0.8

Example 1

The first type of heat radiator is considered (Figure 3a). The geometry of the cross section and the boundary conditions are shown in Figure 4a. This problem is solved by the minimization of the volume of the structure (1). On the edge of each fin the force P equal to $10N$ is assumed. The radiator dissipates $80W$, so applied heat flux on the bottom side depends on the width of the radiator. The ambient temperature is 25^oC, heat convection coefficient is $2W/m^2K$ and emissivity is 0.8. Five design variables are assumed. The method of modeling the shape of the radiator is shown in Figure 4b.

Several tests have been performed with imposed constrains on the maximal value of equivalent stress $\sigma_{eq}^{ad} = 20MPa$, and the maximal value of the temperature equal to

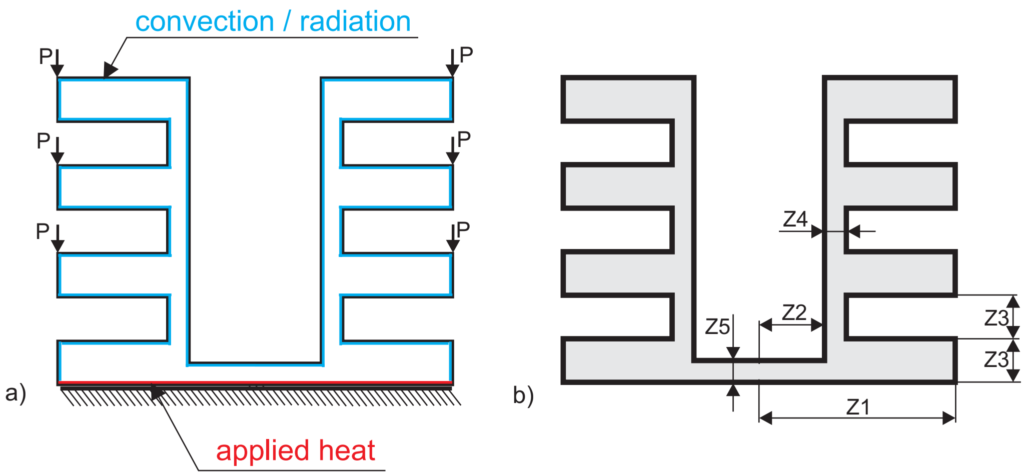


Figure 4. a) The geometry and the boundary conditions of the heat radiator, b) The design variables

70^oC, 80^oC, 90^oC respectively. Table 2 contains the admissible values of the design parameters, whereas Table 3 and Figure 5 shows the result of the optimization.

Table 2. The admissible values of the design parameters

Design variable	Range
Z1	$20mm \div 100mm$
Z2	$2mm \div 10mm$
Z3, Z4, Z5	$4mm \div 10mm$

Table 3. The result of the optimization

	Z1	Z2	Z3	Z4	Z5	Volume
$T^{ad} = 90°C$	$24.19mm$	$10mm$	$4.129mm$	$4mm$	$4mm$	$10367mm^3$
$T^{ad} = 80°C$	$31.42mm$	$10mm$	$4mm$	$4.844mm$	$4mm$	$14107mm^3$
$T^{ad} = 70°C$	$41.46mm$	$9.817mm$	$4mm$	$5.698mm$	$4mm$	$19649mm^3$

Example 2

The problem of the optimal shape of the second type of a radiator is considered (Figure 3b). The geometry of the cross section, fixed dimensions (in mm) and boundary conditions are shown in Figure 6a. The values of boundary conditions are presented in Table 4. The thickness of the structure is $0.2m$.

This problem is solved by the minimization of three proposed functionals (1)(2)(3). In the case of the minimization volume of the structure the constrain on the maximal value of equivalent stress $\sigma_{eq}^{ad} = 15MPa$ and maximal value of temperature in the structure $T^{ad} = 70°C$ are applied, whereas for the minimization of the maximal value of the

Figure 5. The optimal shape of the radiator for: a) $T^{ad} = 90°C$, b) $T^{ad} = 80°C$, c) $T^{ad} = 70°C$

Table 4. The values of the boundary condition

boundary conditions	value
heat flux	$1000W/m^2$
heat convection coefficient	$2W/m^2K$
ambient temperature	25^oC
emissivity	0.8
pressure	$5000Pa$

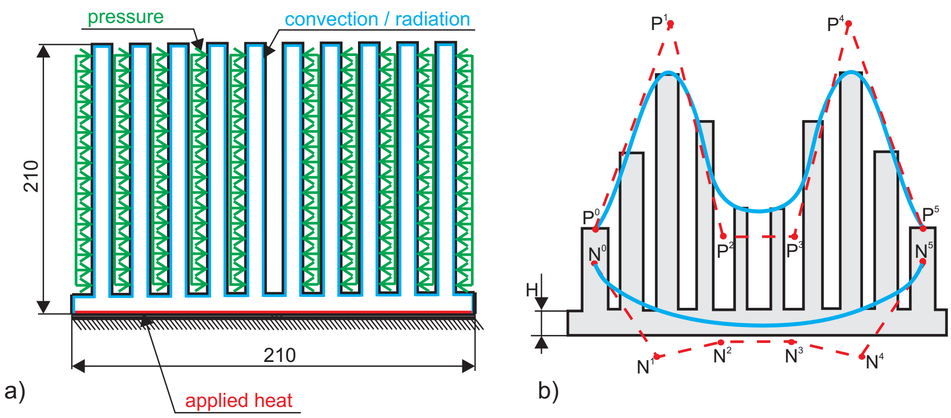


Figure 6. a) The geometry and the boundary conditions of the heat radiator, b) The method of modeling the shape of radiator

equivalent stress and temperature the constrain on the maximal volume of the structure $V^{ad} = 150000mm^3$ is applied.

The constant number of fins, equal to ten, is assumed. The height and width of the fins can vary during the optimization process. It is modeled using Bezier curves consisting of 6 control points. The control polygon of the height (P^0 - P^5) and control polygon of the width (N^0 - N^5) of the fins are shown in the Figure 6b The values of the control points P^0 - P^5 are responsible for the shape of the radiator, whereas the values of the control points P^0 - P^5 for the width of the fins The height of the bottom part of the structure can also vary. Due to symmetry ($\mathrm{P}^0 \stackrel{sym}{\longleftrightarrow} \mathrm{P}^5$, $\mathrm{P}^1 \stackrel{sym}{\longleftrightarrow} \mathrm{P}^4$, $\mathrm{P}^2 \stackrel{sym}{\longleftrightarrow} \mathrm{P}^3$, $\mathrm{N}^0 \stackrel{sym}{\longleftrightarrow} \mathrm{N}^5$, $\mathrm{N}^1 \stackrel{sym}{\longleftrightarrow} \mathrm{N}^4$, $\mathrm{N}^2 \stackrel{sym}{\longleftrightarrow} \mathrm{N}^3$) the total number of the design parameters is equal to 7. The admissible values of the design parameters are shown in Table 5.

Table 5. The admissible values of the design parameters

Design variable	Range
P^0 , P^1 , P^2 , P^3 , P^4 , P^5	$30mm \div 200mm$
N^0 , N^1 , N^2 , N^3 , N^4 , N^5	$4mm \div 12mm$
H	$7mm \div 15mm$

Several numerical tests have been performed for each case. The best results of the optimization are presented in Table 6 and Figure 7.

Table 6. The result of the optimization

	$\min_{X} T^{\max}(X)$	$\min_{X} V(X)$	$\min_{X} \sigma_{eq}^{\max}(X)$
$P^0 = P^5$	$200mm$	$110.6mm$	$80.5mm$
$P^1 = P^4$	$99.13mm$	$30mm$	$57.1mm$
$P^2 = P^3$	$138.9mm$	$30mm$	$71.3mm$
$N^0 = N^5$	$4.49mm$	$4.2mm$	$11.4mm$
$N^1 = N^4$	$4mm$	$4mm$	$5.6mm$
$N^2 = N^3$	$4mm$	$4mm$	$10.3mm$
H	$7mm$	$7mm$	$8.85mm$
Fitness function value	49.48^oC	0.0073	$0.97MPa$

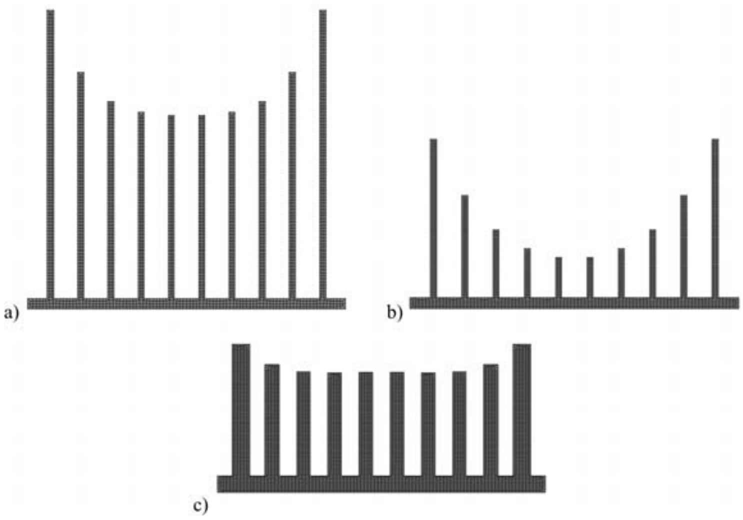


Figure 7. The optimal shape of the radiator: a) minimization of the maximal value of the temperature (3) b) minimization of the volume of the structure (1) c) minimization of the maximal value of the equivalent stress (2)

6 Conclusions

An effective intelligent technique of evolutionary design based on parallel computation has been presented. The important feature of this approach is its great flexibility and the strong probability of finding the global optimal solution. The parallel evolutionary algorithm allows short optimization time. Bezier curves allow the number of design parameters to be reduced.

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