Evolutionary Optimization and Identification of Hybrid Laminates

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Abstract. The paper deals with the optimization and identification of hybrid fibre-reinforced laminates. Two tasks are considered: i) the ply orientations optimization, ii) the material constants identification. Different optimization and identification criteria connected with the modal analysis are considered. The evolutionary algorithm is used as the optimization and identification method. In order to accelerate the calculations, the distributed version of the evolutionary algorithm is employed. Finite element method software is used to solve the direct problem for the laminates. Numerical examples of optimization and identification are attached.

1 Introduction

Composites are materials which play a significant role in modern industry. They are constructed by joining two or more materials together if the union is at the macroscopic level. They usually consist of two phases: the matrix and the reinforcement. The composites that are especially popular are the group of composites that are fibre-reinforced and made of many layers (laminas, plies) ones, called laminates. The laminas are assumed to be joined permanently. The fibres are typically situated directionally in each ply of the laminate. In the one-directional laminates the direction of the fibres in plies is the same. In multidirectional laminates the direction of the fibres in plies is different.

There are two main reasons for the popularity of laminates:

1. The high weight-strength ratio in comparison with the conventional materials.
2. The ease with which the material properties can be tailored to requirements by manipulating the laminate parameters such as: components materials, stacking sequence, fibre orientations and layer thicknesses.

In the most practical laminate applications only the fibre directions in plies and ply thicknesses are different - the material remains the same. Unfortunately, the cost of laminates rapidly increases with their strength. To avoid this inconvenience, laminates can be composed of more than one material. Hybrid laminates usually consist of two materials: the core layers are built of a weaker and less expensive material and the external layers are made of a high-stiffness and more expensive material. This attitude ensures the high efficiency of the laminate reducing its total cost [3].

Composites are generally anisotropic materials. Multilayered laminates can be usually treated as orthotropic materials. If the plies are distributed symmetrically to the mid-plane, the laminate is called symmetrical. The single ply of the laminate, assuming the plane-stress state, has 4 independent material constants [10]: axial and transverse Young's module (E_1, E_2), axial-transverse shear modulus (G_{12}) and axial-transverse Poisson ratio (ν_{12}).

The aim of the present paper is the optimization and the identification of laminate structures. The optimization is performed to achieve the desired properties of the material manipulating the ply orientations. The identification goal is to determine elastic constants of the laminates as well as material densities. The symmetric, hybrid fibre-reinforced laminates are considered. The dynamic behavior of the structures is considered and the modal analysis is employed to obtain data for both tasks. The continuous and discreet versions of the optimization tasks are considered.

To solve the optimization and the identification tasks the evolutionary algorithm is used as the global optimization procedure. To decrease the computation time, the distributed version of the evolutionary algorithm is employed. The finite element method software package is used to solve the direct problem for the laminate plates. The numerical examples are attached.

2 Modal Analysis of Hybrid Laminates

Dynamic properties of the structure can be determined by means of modal analysis both theoretically and experimentally [13]. Theoretical analysis is usually performed by means of the numerical methods, especially the finite element method. Experiments consist in the excitation of the real structure and measurements of the response (displacements, velocities or accelerations at the sensor points) characterizing dynamical behavior of the structure.

The eigenfrequency problem for plates can be presented in the general form:

$$\left(\mathbf{K} - \omega^2 \mathbf{M}\right) \mathbf{u} = 0 \tag{1}$$

where: $\mathbf{K}$ - stiffness matrix; $\mathbf{M}$ - mass matrix; $\mathbf{u}$ - eigenfunction (displacement vector) corresponding to the eigenvalue ω.

For a rectangular hybrid laminate plate of length a, width b and thickness h in directions x, y and z, respectively, the previous equation has the form [4]:

$$\rho h \omega^2 w = D_{11} w_{,xxxx} + D_{16} w_{,xxxy} + 2(D_{12} + 2D_{16}) w_{,xxyy} + 4D_{26} w_{,xyyy} + D_{22} w_{,yyyy} \tag{2}$$

where: w - deflection in the z direction, D_{ij} - bending stiffness, ρ - mass density.

The relatively small data set of eigenvalues can result in equivocal results of identification. To get more data having a small set (typically one) of sensor points the frequency response analysis can be used. Accelerometers are especially convenient because: i) they have relatively small mass compared to displacement and velocity sensors; ii) it is possible to obtain velocity and displacement signals by the integration of accelerations signal.

The dynamic equilibrium equations for a structure subjected to harmonic excitation has the form:

$$\mathbf{K}\mathbf{u} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{M}\ddot{\mathbf{u}} = \mathbf{P} \sin \omega t \tag{3}$$

where: $\mathbf{C}$ - damping matrix, $\mathbf{P}$ - excitation force vector, $\mathbf{t}$ - time, ω - frequency, $\dot{\mathbf{u}}$ - velocity vector, $\ddot{\mathbf{u}}$ - acceleration vector.

3 Optimization Task

The objective of the optimization task is to find the optimal set of ply angles for given criteria connected with the eigenfrequencies. To achieve that, the objective function $J_0(\mathbf{x})$, where $\mathbf{x}$ is a vector of the design variables, is maximized:

$$\max : J_0(\mathbf{x}) \tag{4}$$

The ply orientations are typically restricted to a small set of discrete angles (0°, +45°, -45°, 90°) due to the manufacturing process. There exist tow placements machines able to produce laminate with arbitrary ply angles, but they are expensive and not popular. To take the foregoing fact into consideration discrete as well as continuous variants of the optimization tasks are examined.

The number and thicknesses of the laminas of a hybrid laminate are assumed constant. The symmetric, fibre-reinforced hybrid laminates made of two materials are examined. The plies orientations (angles) are the design variables. Due to the symmetry, the number of design variables is a half of the plies number.

The $\mathbf{x}$ vector has the form:

$$\mathbf{x} = (\varphi_1, \varphi_2, ..., \varphi_i, ..., \varphi_n) \tag{5}$$

where: φ_i - i-ply orientation angle; n - a half of the plies number.

The dynamic characteristics analysis of the structure by means of the modal analysis is employed to solve the considered laminate's optimization task. Two typical optimization criteria connected with the free vibrations of structures are considered:

1. The maximization of the distance between two first eigenfrequencies:

$$\max : [J_0(\mathbf{x}) = \omega_2(\mathbf{x}) - \omega_1(\mathbf{x})] \tag{6}$$

2. The maximization of the minimum distance between the external excitation frequency ω_{ex} and the eigenfrequency ω_i:

$$\max : [J_0 = \min(|\omega_i(\mathbf{x}) - \omega_{ex}(\mathbf{x})|)] \tag{7}$$

4 Identification Task

The objective of the identification task is to find the material constants in a multilayered, hybrid, fibre-reinforced laminate. Due to the fact, that the laminates are anisotropic materials, it is often necessary to identify the elastic properties of the designed element. Laminate structures are frequently manufactured as the unique ones, so the non-destructive tests have to be performed [8]. The identification task belongs to inverse problems, where unknowns (material properties, geometry, boundary conditions etc.) are recognized on the basis of the responses to given excitations measured on its boundary [5]. Indirect identification methods, usually based on the numerical and mixed numerical-experimental methods are intensively developed [11]. Identification problems are the ill-posed ones and the identification results can be ambiguous if there is insufficient measurement data.

The problem of the elastic constants identification for multi-layered laminates made of one material was previously solved successfully by authors [7]. The static data (displacements at sensor points) as well as the modal analysis data (eigenfrequencies and frequency response in one sensor point) were used for identification.

Identification can be treated as the minimization of the objective function J_0 with respect to the vector of the design variables $\mathbf{x}$:

$$\min : \left[J_0(\mathbf{x}) = \sum_{i=1}^{N} \left| \frac{\hat{\mathbf{q}}_i - \mathbf{q}_i}{\hat{\mathbf{q}}_i} \right| \right] \tag{8}$$

where: $\mathbf{x} = (x_i)$ - the parameters representing the identified constants; $\hat{\mathbf{q}}_\mathbf{i}$ - measured values of a state fields; $\mathbf{q}_\mathbf{i}$ - values of the same state fields calculated from the solution of the boundary-value problem.

The numbers of layers, their thicknesses and fibre orientation are assumed to be known. It is assumed that external layers of the laminate are made of material 1 and the core layers are made of material 2 and the number of layers made of each material is known. The $\mathbf{x}$ vector has the form (with superscripts specifying the material number):

$$\mathbf{x} = \left(E_1^1, E_2^1, G_{12}^1, \nu_{12}^1, \rho^1, E_1^2, E_2^2, G_{12}^2, \nu_{12}^2, \rho^2 \right) \tag{9}$$

The direct problems for optimization as well as for the identification tasks are solved by means of a professional finite element method software package (MSC.PATRAN/ NASTRAN) with laminate modeler [1].

5 Distributed Evolutionary Algorithm

In order to identify the material constants and achieve desired properties of laminates, optimization methods are applied.

In the present paper the evolutionary algorithm (EA) is employed to solve both tasks. EA is especially useful in two cases: i) if gradient methods fail due to the fact that the information about the objective function gradient is hard or impossible to obtain; ii) if objective function is multimodal, which usually leads the gradient methods to the local optima. The only information the EA needs to work is the objective (fitness) function value [6].

Considered optimization and identification tasks are usually multimodal ones. In addition, the ply angles in the presented optimization task are typically restricted to a relatively small set of discrete angles because of manufacturing limitations.

One of the disadvantages of the EA is time-consuming computation. This fact is especially noticeable in engineering problems, in which the solving of the direct problem is the most expensive part of calculations. To reduce this inconvenience the distributed EA (DEA) is exploited [9]. The total population of chromosomes is divided into two or more subpopulations. Subpopulations evolve nearly independently. Information between subpopulations is interchanged during a migration phase. Due to the fact, that sending chromosomes to processors is managed by a special process, the number of processing units can in total be up to the number of chromosomes. The block diagram of the evolutionary algorithm (for one subpopulation) is presented in Figure 1.

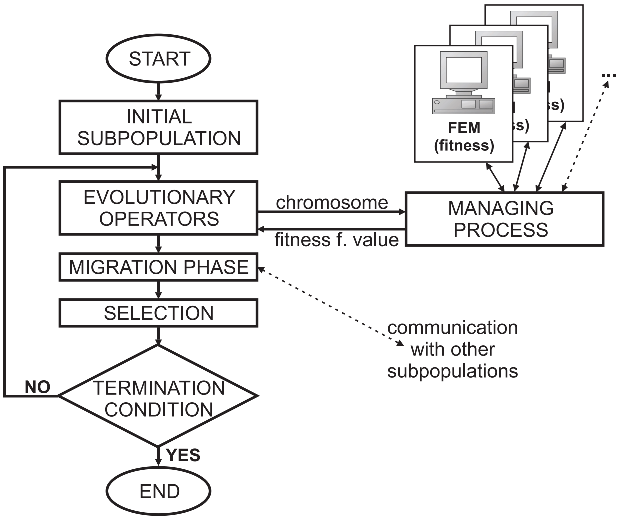

Figure 1. The block diagram of the DEA - one subpopulation.

6 Numerical examples

A rectangular, symmetric, hybrid laminate plate is examined (Figure 2).

The external plies of the laminate are made of material M_1, the core plies are made of the material M_2. The material properties [12] are collected in Table 1. The plate FEM model is divided into 200 4-node elements of type QUAD4. The genes (design variables) are the floating-point numbers in continuous cases and the whole numbers in the discrete cases. The population of chromosomes is divided into 2 supbopulations in each case. Rank selection with elitism is used as the selection method.

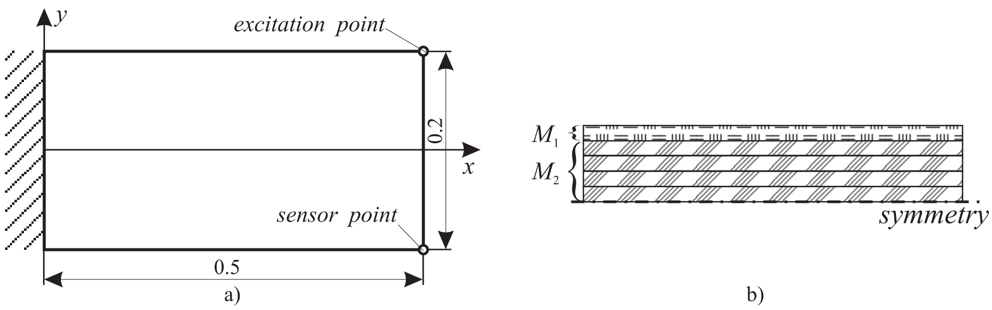

Figure 2. The hybrid laminate plate: a) dimensions and bearing; b) location of materials and symmetry plane (10 plies).

Table 1. The laminate materials - parameters.

Material	E_1 [GPa]	E_2 [GPa]	G_{12} GPa]	ν_{12}	ρ [kg/m^3]
M_1(graphite-epoxy T300/5280)	181	10.3	7.17	0.28	1600
M_2 (glass-epoxy, Scotchply 1002	38.6	8.27	4.14	0.26	1800

6.1 Optimization of the Laminate

Two optimization cases are considered: 10-plies and 20-plies. To compare the results the thicknesses of the parts made of particular materials are the same in both cases (so the total thicknesses of laminates are the same as well). The continuous and three discrete variants are taken into account. In the continuous variant each ply can vary continuously in the range [-90°, 90°]. In the discreet variants each ply angle can vary in the range [-90°, 90°] taking values every 5°, 15° and 45°. Two optimization criteria: (6) and (7) are applied. The initial stacking sequences for 10-plies and 20-plies laminates are: (0/15/-15/45/-45)s and (0/0/15/15/-15/-5/45/45/-45/-45)s, respectively. The first 5 eigenfrequencies of the plates are calculated by the FEM software and presented in Table 2.

Table 2. The initial laminate plate - the values of the first 5 eigenfrequencies.

ω_1 [Hz]	ω_2 [Hz]	ω_3 [Hz]	ω_4 [Hz]	ω_5 [Hz]
99.7316	252.9872	582.1273	622.4944	909.4683

The remaining parameters of the DEA are:
i) chromosomes in each subpopulation $N_e = 20$;
ii) evolutionary operators: simple crossover $p_c = 0.9$, uniform mutation $p_{mu} = 0.1$ and Gaussian mutation $p_{mg} = 1/(individual\ length)$.

The results for the maximization of the distance between the first and the second eigenfrequencies are collected in Table 3.

Table 3. The optimization results - 1$^{\text{st}}$ criterion.

Variant	Plies	Stacking sequence	ω_1, ω_2 [Hz]	$\omega_2-\omega_1$ [Hz]
initial	10	(0/15/-15/45/-45)s	99.732, 252.987	153.2556
	20	(0/0/15/15/-15/-15/45/45/-45/-45)s	99.732, 252.987	153.2556
conti-	10	(35.87/-31.52/-32.13/-32.16/31.97)s	60.538, 342.667	282.1298
nuous	20	(34.05/-37.12/-31.35/-19.54/31.87/	67.109, 408.484	341.3746
		27.65/58.04/-34.19/-22.97/-61.09)s		
5°	10	(35/-35/30/-30/-30)s	61.048, 342.868	281.8201
	20	(35/-35/-30/25/30/35/-30/55/35/30)s	67.334, 408.931	341.5970
15°	10	(30/-45/-30/-30/30)s	64.696, 340.285	275.5891
	20	(30/-30/45/-45/45/-45/-45/45/-45/-45/)s	71.076, 398.048	326.9711
45°	10	(45/-45/0/0/0)s	51.825, 310.238	258.4137
	20	(45/-45/0/0/0/0/0/0/0/0)s	61.922, 380.129	318.2076

For the second criterion it was assumed, that the excitation frequency is constant and equals ω_{ex}=120Hz. The distance to the closest eigenfrequency is maximized. The optimization results are collected in Table 4.

Table 4. The optimization results - 2^{nd}criterion.

Variant	Plies	Stacking sequence	ω_1, ω_2 [Hz]	min($\|\omega_i$-$\omega_{ex}\|$) [Hz]
initial	10	((0/15/-15/45/-45)s	99.732, 252.987	20.268
	20	(0/0/15/15/-15/-15/45/45/-45/-45)s	99.732, 252.987	20.268
conti-nuous	10	(76.87/88.87/61.13/6.11/61.30)s	33.126, 206.874	86.874
	20	(80.42/-76.28/61.97/87.51/-48.12 /71.62/12.09/53.8/85.88/45.63)s	33.127, 206.875	86.873
5°	10	(90/60/-45/50/90)s	33.155, 207.095	86.845
	20	(-80/90/65/55/65/25/-65/-85/85/15)s	33.120, 206.880	86.880
15°	10	((-75/90/-60/15/-15)s	33.260, 207.019	86.739
	20	(90/75/45/90/60/-60/-45/90/-30/90/)s	33.188, 207.031	86.815
45°	10	(90/-45/90/45/90)s	33.639, 206.624	86.3612
	20	(90/90/90/45/45/45/90/90/90/90)s	33.198, 207.350	86.802

6.2 Identification of the Laminate

The plate was excited in the right-upper corner by the sinusoidal signal. The frequency of the excitation varied from 100Hz to 20000Hz with a step of 100Hz. 200 samples of the acceleration amplitudes at one sensor point situated in the right-lower corner of the plate were measured.

The remaining parameters of the DEA are:
i) chromosomes in each subpopulation $N_e = 50$;
ii) evolutionary operators: simple crossover $p_c = 1.0$ and Gaussian mutation $p_{mg} = 1/(individual\ length)$.

The number of plies, ply thicknesses and fibre direction in each ply are assumed to be known. The identification was performed for 20-plies, symmetric laminate of the stacking sequence: (0/0/15/15/-15/-5/45/45/-45/-45)s.

The identification results are collected in Table 5. Frequency response diagram is presented in Figure 3.

Table 5. The identification results.

		Identified constant				
Material		E_1 [GPa]	E_1 [GPa]	G_{12} [GPa]	ν_{12} [GPa]	ρ_{12} [kg/m^3]
M1 (outer)	exact value	181.00	10.30	7.17	0.280	1600.0
	found value	183.81	10.35	7.63	0.2683	1896.4
	error [%]	1.55	0.44	6.42	4.18	18.53
M2 (inner)	exact value	38.60	8.27	4.14	0.26	1800.0
	found value	34.96	8.65	3.93	0.231	1725.32
	error [%]	9.44	4.61	5.04	11.38	4.15

6.3 Computing Speed-Up

The computation speedup obtained by a dual-core processor was investigated. The speedup of computation s can be expressed as the time necessary to solve a problem on

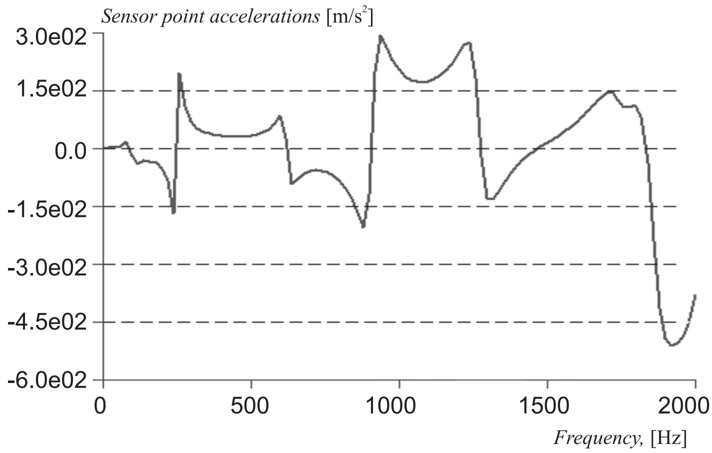

Figure 3. Accelerations at the sensor point - the frequency response.

one processing unit t_1 over a solution time on n processing units t_n:

$$s = \frac{t_1}{t_n} \tag{10}$$

The identification of material constants in hybrid laminate was used as a benchmark problem. The tests were performed using the DPEA algorithm.

The dual-core Intel Pentium D830 processor was employed. Intel dual-core processors have two complete processor cores in one physical package running at the same frequency. The cores share the same interface with chipset and memory, but each core has its own set of registers and cache [2].

The average fitness function evaluation time using one core was about one second. The 1.7 speedup was obtained during the tests. The speedup is lower than linear because it is restricted by the common, single processors interface to the memory.

7 Final Conclusions

Hybrid laminates, being not as popular as the laminates with laminas made of the same composite, ensure high efficiency with a lower cost of the laminate. The stacking sequence optimization gives the possibility to obtain the required properties of the laminate for given criteria. Non-destructive methods of the identification of the material constants in laminates are especially important due to their frequent unitary production.

Coupling the finite element method with the evolutionary algorithm gives interesting alternative for traditional optimization and identification methods, especially for discrete and multimodal problems, encountered in the considered issues.

The modal analysis has been employed for both tasks. The distance between two consecutive eigenfrequencies as well as the gap between the excitation frequency and the nearest eigenfrequency were significantly enlarged. The frequency response analysis gives large amount of measurements in a small number (only one in the present case) of sensor points. The identification results are not quite precise, but promising. Coupling frequency response with the eigenfrequency data would improve the identification results, and wil be tested in further research.

For the real problems the computation of the fitness function by means of the finite element method is the most time-consuming part of calculations. It can be significantly reduced by means of the distributed EA.

8 Acknowledgements

The research is financed from the Polish science budget resources in the years 2006-2007 as the research project.

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