On some attempts of Evolutionary Algorithms efficiency increase

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1. Introduction

An approach to residual stress analysis using evolutionary algorithms (EA) is considered. The final objective of this research is residual stress analyses in railroad rails and vehicle wheels. These residual stresses are of great technical importance for reliable prediction of safe rail and/or wheel life service. Both theoretical and experimental investigations of these stresses present complex large numerical tasks formulated in terms of non-linear constrained optimization problems. So far several discrete methods (FE, BE, MFD) and deterministic solution approach were used [1] to solve these problems, but neutral networks (NN) were tried as well [2]. An attempt has very recently been made [3,4], and is currently continued here [2] in order to propose and examine the EA solution approach. However, due to the optimization problem in question, its size and complexity, special attention should be paid first to an essential increase in the efficiency of the EA applied. Several new concepts were proposed and are examined here using a variety of simple, but large size 1D and 2D tests (up to 2000 decision variables) to analyze residual stress generated in a prismatic bar subject to pure elastic-plastic bending. The following speed-up factors were achieved by means of the concepts mentioned above: ~2, ~3, ~10, while a simultaneous application of all of them resulted in the speed-up factor of up to twenty thus far. It is worth noticing that the closer to the exact solution one is, the larger the increase in EA efficiency. Further research is planed including continuation of efforts oriented increasing the efficiency of EA, analysis of residual stresses in true railroad components and the analysis of related large non-linear non-convex optimization problems.

2. Formulation of optimization problems to be solved by the EA approach

As it was already mentioned the theoretical as well as hybrid theoretical experimental analysis of residual stresses in railroad rails present complex tasks formulated in terms of non-linear constrained optimization problems. The shake-down mechanical model for evaluation of residual stresses in elastic-perfectly plastic material proposed in [5], was extended for linearly strain hardening material [6,7]. It is formulated in two steps as follows:

(i) calculate the correlation matrix A_{ijkl} $$\sigma_{ij}^{r} = A_{ijkl} \cdot \varepsilon_{kl}^{p} \qquad (2.1)$$

solving the following nonlinear constrained optimization problem for self equilibrated stress σ_{ij}^{r} as a function of plastic distortions ε_{ij}^{p} :

$$\text{find:} \quad \min \Theta\left(\sigma_{ij}^{r}\right) \quad ,\Theta\left(\sigma_{ij}^{r}\right)=\int_{V} \sigma_{kl}^{r} \cdot C_{ijkl} \cdot \varepsilon_{ij}^{r} \cdot dV - \int_{Vij} \varepsilon_{ij}^{p} \cdot \sigma_{ij}^{r} \cdot dV \tag{2.2}$$

satisfying the conditions

$\sigma_{ij,j} = 0$ in V – internal equilibrium conditions

$\sigma_{ij,j} \cdot n_j = 0$ on ∂V – static boundary conditions;

(ii) find ε_{ij}^{rp} which minimize the total complementary energy functional $\psi(\varepsilon_{ij}^{p})$:

$$\min_{\varepsilon_{ij}^{p}} \Psi\left(\varepsilon_{ij}^{p}\right) \quad ,\Psi\left(\varepsilon_{ij}^{p}\right)=\int_{V} \varepsilon_{ij}^{p} \cdot A_{ghij}^{p} \cdot C_{ijkl} \cdot A_{klmn} \cdot \varepsilon_{mn}^{p} \cdot dV \tag{2.3}$$

satisfying the yield condition $\quad \Psi\left(\left(A_{ghij} - I_{ghij} \cdot c\right) \cdot \varepsilon_{ij}^{p} + \sigma_{ij}^{E}\right) - \sigma_{Y} \leq 0 \quad$ (2.4)

hence the residual stress $\quad \sigma_{ij}^{r} = A_{ijkl} \cdot \varepsilon_{kl}^{rp} \quad$ (2.5)

where $c = \dfrac{E \cdot H}{E - H}$ hardening parameter; C_{ijkl} – elastic compliance matrix; E – Young modulus; H – strain hardening modulus; I_{ghij} – unit matrix; ε_{ij}^{p} – residual plastic strain; σ_{ij}^{r} – estimated residual stresses in a body under consideration due to actual applied loads; σ_{ij}^{E} – elastic stresses computed as if the object behavior was purely elastic during the loading process; σ_{Y}^{E} – yield stress.

The above presented formulation is one of the target research problems that are to be solved in order to determine distribution of stresses throughout the rail or wheel cross-section. However, in the present paper a benchmark problem is dealt with, namely the search for residual stresses in an elastic perfectly plastic bar of rectangular cross-section (bx2H) subject to cyclic bending by the moment M exceeding its elastic capacity. This formulation is given below together with the relevant discrete form used to apply the EA.
Find self-equilibrated normal stresses $\sigma(x,y)$ that minimize complementary energy of the beam:

$$\min_{\sigma(x,y)} \int_{-b}^{b} \int_{0}^{H} \sigma^{2} dxdy \tag{2.6}$$

satisfying global condition of equilibrium for bending: $\quad 2b\int_{0}^{H} \sigma \, y \, dy = 0 \quad$ (2.7)

and inequality conditions resulting from the yield criterion: $\quad \left|\sigma + \sigma^{e}\right| \leq \sigma_{Y} \quad$ (2.8) where: $$\sigma^e = \frac{3}{2}\sigma_Y\left[1-\frac{1}{3}\left(\frac{\bar{y}}{H}\right)^2\right]\frac{y}{H}\cdot\sigma_Y \tag{2.9}$$

is the normal stress due to the bending of the considered bar as the purely elastic body; $\bar{y}$ - limit of the elastic zone in the true elastic-perfectly plastic bar; σ_y - yield limit.

Introducing a rectangular mesh in the beam cross-section, and assuming unknown stress value $\sigma_{k,}$ in each node for $k=1,\ldots,N$ one obtains corresponding discrete formulation of the above optimization problem. It will be solved using the EA.

3. The EA solution approach to optimization problems

3.1. The EA solution approach

The following version of the solution approach using the EA was chosen and tested:

- tournament selection,
- simple crossing over applied alternately with heuristic one
- unequal mutation.

Population within the range of 100÷1000 chromosomes was used while number of decision variables in the chromosome (stress values) varied, up to 2000.

3.2. On acceleration of the EA solution process

One of the most significant drawbacks of genetic and evolutionary algorithms lies in time consuming solution process when compared with relevant deterministic methods. Optimization problems formulated above for residual stress analysis may be characterized as large tasks. Therefore, the efficiency of EA algorithms presents a crucial point for their successful solution. There are various classical ways, of increasing their efficiency including tracing the best chromosomes in population, scaling of the fitness function or using EA with built-in information on the fitness function gradients. However, in the present paper we would like to present and test several original concepts for essential speeding-up the EA calculations. The following concepts were considered:

i) various types of smoothing followed by global balancing of the raw stress results provided by the EA analysis;

ii) non-standard use of parallel and distributed calculations carried out on a cluster, in order to obtain fast solution improvement;

iii) concentration of analysis in zones of large errors; a posteriori error estimation has been proposed based on averaged solutions obtained by means of (ii), and error level has been used to influence selection probability.

These concepts are outlined and presented below. They were successfully tested on a variety of benchmarks.

3.3. Smoothing and balancing of raw EA solution

Raw results obtained from the EA approach present a collection of locally scattered data while the true solutions usually are smooth at least piecewise. Therefore, an attempt has been made to apply moving weighted least squares (MWLS) technique [8] in order to smoothen the raw results of the standard EA approach. In the basic MWLS version one minimizes at each point the weighted error functional

$$B = \sum_{i=1}^{n} \left(w_i - \tilde{w}_i \right)^2 \cdot v_i^2 \tag{3.1}$$

with respect to the set of local derivatives of function w .

Here w_i is a function value supplied by the EA, $\tilde{w}_i$ presents its approximation by means of expansion into the p-th order truncated Taylor series, while v_i^2 is a weighting factor.

In the case of the second order 2D approximation used here (p=2) one has

$$\begin{aligned} \tilde{w}_i &= w_i \left(x + h_i, y + k_i, \right) \\ &\approx w + h_i \cdot w_{,x} + k_i \cdot w_{,x} + \frac{1}{2} \cdot h_i^2 \cdot w_{,xx} + \frac{1}{2} \cdot k_i^2 \cdot w_{,yy} + k_i \cdot h_i \cdot w_{,xy} \end{aligned} \tag{3.2}$$

$$h_i = x - x_i \quad , k_i = y - y_i \quad \text{,and} \quad v_i = \left(\rho_i^2 + \frac{g^4}{\rho_i^2 + g^2} \right)^{-p-1} , \quad \rho_i^2 = h_i^2 + k_i^2 \tag{3.3}$$

where g is a smoothing parameter. Minimization conditions:

$$\frac{\partial B}{\partial w} = 0; \quad \frac{\partial B}{\partial w_{,x}} = \frac{\partial B}{\partial w_{,y}} = 0; \qquad \frac{\partial B}{\partial w_{,xx}} = \frac{\partial B}{\partial w_{,xy}} = \frac{\partial B}{\partial w_{,yy}} = 0 \tag{3.4}$$

provide a set of simultaneous linear equations to be solved for unknown function value w and its derivatives $w_{,x}$, $w_{,y}$, $w_{,xx}$, $w_{,xy}$,, $w_{,yy}$ at the point (x,y), where w has been expanded into Taylor series.

Appropriate choice of the smoothing parameter $g \geq 0$ is of significant practical importance. For g=0 one deals with singularity of the weighting factor v_i providing in this way interpolation for all given w_i data (i=1,...,N). Otherwise, the higher the value of parameter g , the smoother the approximation. Each smoothing as a side effect results in global equilibrium loss of the considered body (a bar here). The equilibrium is restored after a series of the EA steps. This process, however, may be aided by artificial balancing done immediately after each smoothing. In the bar cross-section F the resultant, unbalanced axial force is evaluated, and relevant static moment is found by using the EA solution based stress data σ.

$$N = \int \sigma \cdot dF \quad , \quad M_X = \int y \cdot \sigma \cdot dF \quad , \quad M_Y = \int x \cdot \sigma \cdot dF \tag{3.5}$$

Assuming now the linear correction term of the form $\sigma = a \cdot x + b \cdot y + c$ (3.6)

and using the same formulas (3.5.) one may equalize the relevant global quantities with unbalanced ones, and find unknown parameters a,b,c. In the case when (x,y) are Cartesian coordinates determined with respect to the central principal axes,

$$a = \frac{M_Y}{I_Y} \quad , b = \frac{M_X}{I_X} \quad , c = \frac{N}{F} \tag{3.7}$$

where I_x ,I_y are the inertia moments of the bar cross-section F.

A series of test problems have been analyzed for the case of the bar subject to pure cycling bending by the moment M. Smoothing and balancing effects have been investigated for varying values of the smoothing parameter g and for variable number n of standard cycles between the smoothed cycles. These tests results are presented in Fig 1, Fig 2. Values g=10 and n=150 have been found to be the most effective. Effect of balancing is described by the data given in Table 1. Solution (smoothed and non smoothed axial residual stress for half of the bar cross section) is presented in Fig 3.

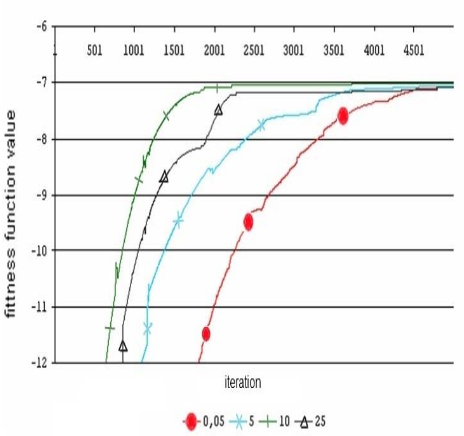


Fig 1 - Smoothing for various values of parameter g

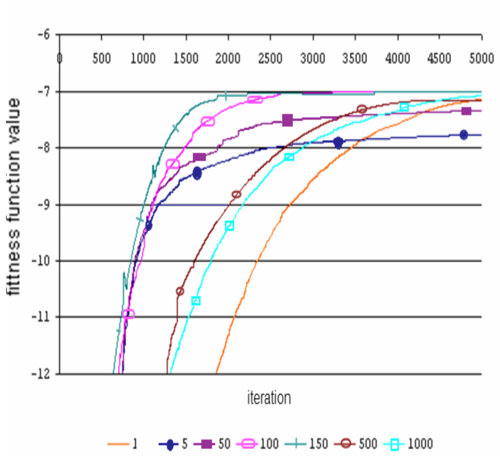
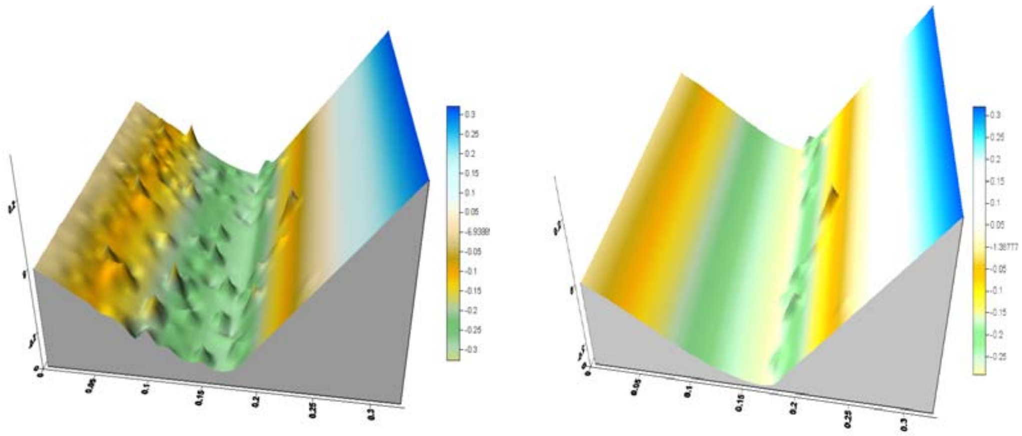


Fig 2 - Smoothing after every n-th iteration, for g=$gopt$=10

Table 1. Effect of the global equilibrium balancing

case	Fitness function value	Error norm	
		maximum	Euclidean
No balancing after smoothing	-10,315	0,23531	0,00674
Balancing after smoothing	-7,023	0,00342	0,00043



non-smoothed solution 2D smoothed solution 2D

Fig 3 - Solution after 6000 iterations

3.4. A` posteriori error estimation

Information on solution error is a necessary part of any reliable analysis. Moreover, advantage may be taken to use this information in order to influence selection probability in those parts of the bar cross-section where solution errors are larger. Such solution strategy is expected to increase efficiency of the EA. Of course, the true errors are not known. However, their a' posteriori estimation is possible.

The main concept of a' posteriori error estimation applied here is to solve the same problem in an independent way m-times. Each of this solutions will be slightly different. The difference between each of them, and the average of m-solutions (the best from each population) may serve as a reasonable error estimate.

The algorithm is as follows. Let us denote i-th solution by:

$$\left\{ z_1^{(i)}, z_2^{(i)}, z_3^{(i)}, \ldots, z_n^{(i)} \right\} \tag{3.8}$$

where z_k^i is k-th chromosome in i-th solution, m is the number of solutions (or processors), n – number of chromosomes (decision variables) in population. The averaged solution is

$$\{z_1^{(av)}, z_2^{(av)}, z_3^{(av)}, \ldots, z_m^{(av)}\} = \frac{1}{m}\sum_i \{z_1^{(i)}, z_2^{(i)}, z_3^{(i)}, \ldots, z_m^{(i)}\} \qquad (3.9)$$

The simple arithmetic average may be replaced by the weighted average where weighting factors depend on the fitness function values.

Thus the unknown true error $z_k^{(i)} - z^{exact}$ is estimated by the approximate one $z_k^{(i)} - z^{(av)}$.

Typical contour maps and 3D picture of true and estimated errors are shown in Fig 4 and Fig 5. Typical results of error aided solution process are presented in Fig 6.

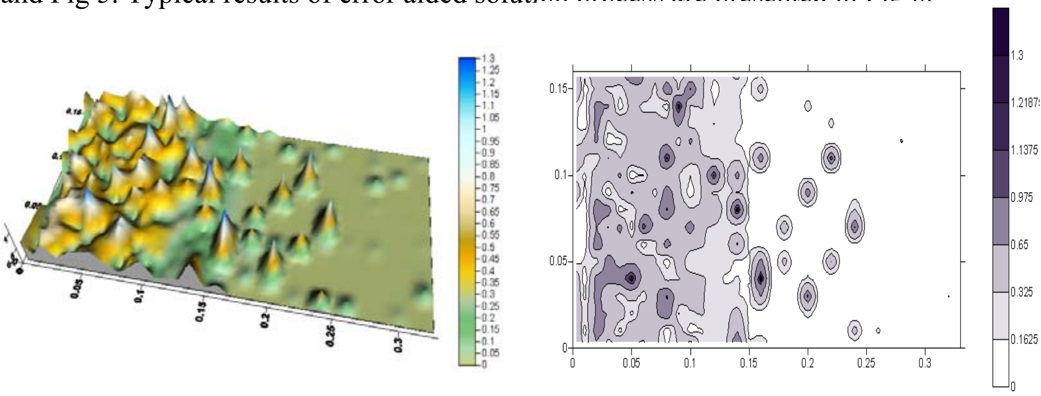


Fig 4 Exact error

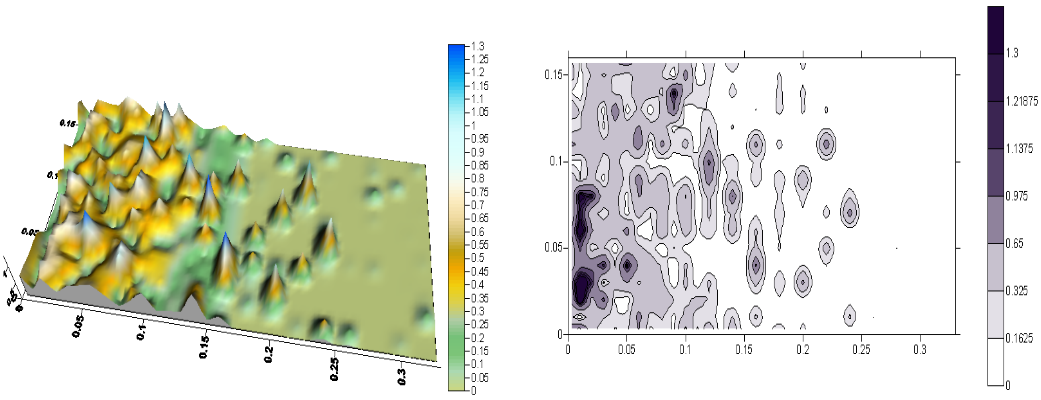


Fig 5 - Estimated error

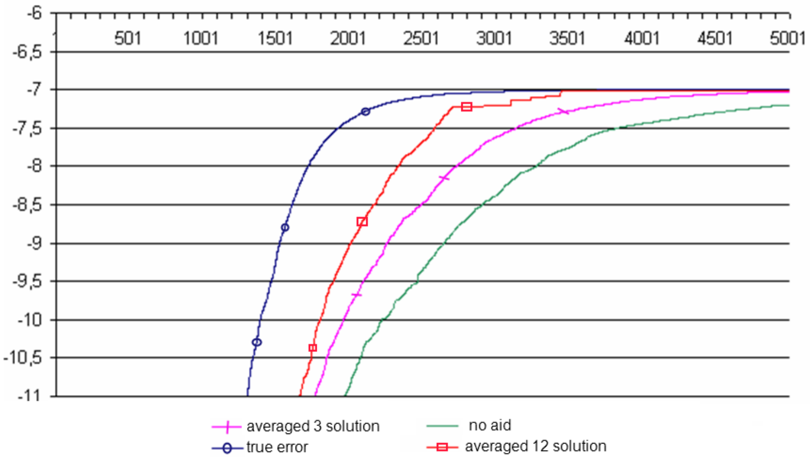


Fig 6 - Solution using: - true error; - averaged 12 solution;
- averaged 3 solution; - no support

3.5. Distributed, and parallel calculation

Time consuming EA analysis may be shortened by means of distributed and/or parallel calculations. A typical approach is based on division of one full task, usually designed for one computer, into many subtasks performed on different processors (computers). These are used at the same time, each one performing only a portion of the whole task.

Such option might be also available here. However, a different, original approach is considered. We use cluster with m-processors. Each one is used to run independent full solution proc
stopped, and follo
processor) sends its best chromosomes to one principal processor. In this way the principal population consists of the 'best solutions' and faster tends towards the final solution. The other advantage of using cluster is calculation of the average obtained for solutions found by all processors, as has been mentioned above. Speed-up based on using 4,8 and 12 processors for the problem discretized by 1200 decision variables, a population of 400 chromosomes and up to 25000 iterations, are presented in Fig 7.

When additional smoothing was applied after 1500-th iteration the maximum and Euclidean error have been found equal to 0,0032 and 0,0008 versus 0,0424 and 0,0028 respectively when no smoothing has been used.

When all speed-up mechanisms have been applied together the total calculation time has been shortened about 20 times (Fig 8.)

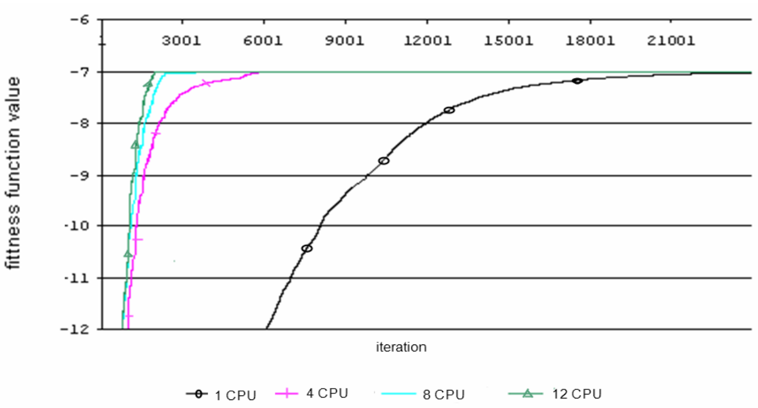


Fig 7 - Display of solution process based on using 1, 4, 8, 12 processors

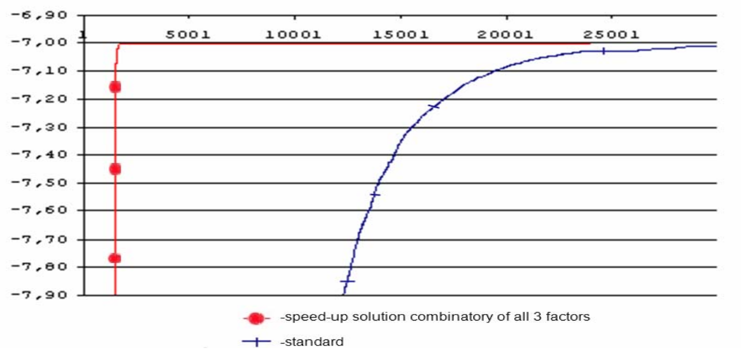


Fig 8 - Comparison of : - standard solution; - accelerated solution by simultaneous combination of all 3 concepts.

4. Final remarks

- -An EA , *efficient* enough to find solution of *large non-linear constrained optimization* problems, similar to those resulting from the analysis of residual stresses in railroad rails has been developed and verified.
- The solution approach was successfully *tested* on several benchmark problems including pure cyclic bending of an elastic plastic bar; optimization problems of up to *2000* decision variables were solved.
- Special emphasis has been laid on approach *efficiency*. Several concepts increasing the solution convergence rate were investigated, including:
  - *concentration* of analysis in zones of *large* estimated *errors* (speed-up factor ~2)
  - raw EA solution *smoothing* and *balancing* (speed-up factor ~3)
  - non-standard distribution of calculations (speed-up factor ~10).

  Combination of the above resulted so far in the speed-up factor increase of at least 20.

It is worth noticing that the closer to the exact solution one is – the larger EA efficiency is noticed.

– Therefore, results currently obtained provide *reasonable hope* for the successful analysis of the true rail problem also.

Further research planned

– Continuation of efforts oriented towards *increasing* the EA *efficiency*, including further parallelization of calculations.
– Analysis of *further benchmarks* including evaluation of residual stresses in thick-walled cylinder subject to combined cyclic loading.
– Reprogramming the EA core in C, so far coded in JAVA.
– Analysis of *residual stresses* in railroad *rails* and vehicle *wheels.*
– Analysis of large *non-linear constrained optimization* problems (convex and non-convex) resulting from the *physically* based *approximation* applied to experimental investigation of residual stresses in railroad components.

5. References

[1] Orkisz J. et al., Development of Advanced Methods for Theoretical Prediction of Shakedown Stress States and Physically Based Enhancement of Experimental Data; Grant Rpt to US DOT FRA, Washington 2004.

[2] Kogut J., Orkisz J., Application of radial basis neutral networks to residual stresses in rail under wandering contact loading, AI-Meth, Gliwice, Nov., 13-15, 2002.

[3] Obrzut A., Analysis of Residual Stresses in Chosen Prismatic Bars by Genetic and Evolutionary Algorithms (in Polish), MSc Thesis, Cracow University of Technology 2005.

[4] Orkisz J., Obrzut A., On Application of the Evolutionary Algorithms to Residual Stress Analysis in Railroad Rails, *Symposium on Method of Artificial Intelligence*, Gliwice, Nov. 16-18, 2005.

[5] Orkisz J. , Prediction of actual stresses by constrained minimization energy, *Residual Stress in Rail, Kluwer* Academic Publishers, Dordrecht, Vol. II, pp 101-124, 1992.

[6] Orkisz J., Cecot W., Prediction of actual residual stresses resulting from cyclic loading in kinematic hardening material, Proceedings of the 5th I*nt. Conf. Computational Plasticity*, Barcelona, Spain, pp. 1029-1042, 1997.

[7] Pazdanowski M., Evaluation of the residual stresses included in a rail by single point contact load. US DOT DTFR53-95-G-000555 Report, 1998.

[8] Orkisz J., Finite difference method (Part III), in Handbook of computational solid mechanics, M. Kleiber (Ed), Springer-Verlag, Berlin, 1998, 336-432