Linear projection procedure in an evolutionary algorithm

Wladyslaw Kaminski and Elwira Tomczak

Technical University of Lodz, Faculty of Process and Environmental Engineering, Lodz, Poland,
e-mail: kaminski@wipos.p.lodz.pl, tomczak@wipos.p.lodz.pl

Abstract. From the view point of practical applications and the theory of evolutionary algorithms, an important problem is the location of many local extrema showing at the same time the position of a global extremum. To accomplish this task, a new space exploration procedure is proposed using a so-called "linear projection". Additionally, a clustering technique is introduced to the selection procedure. Operation of the method is illustrated by searching the function of two variables with extrema separated by a tableland with a box-type limitation. Each time, irrespective of the number of clusters, calculations enabled location of all three extrema.

1 Introduction

The task of any global optimization method is to find a global optimum. The global optimum can be located in one, many or infinite number of points. A global minimum and maximum exist for any continuous function if the set of admissible solutions is both bounded and closed. In this case, a global extremum can occur in the point in which a local extremum appears, or it is located on the boundary. The main disadvantage of evolutionary algorithms is that the local extremum attracts subsequently generated points. This unfavourable phenomenon can be prevented, for instance by an appropriate selection procedure [1]. The problem of leaving the attraction region or a transition through a saddle, was studied extensively, for example, by Chorążyczewski and Galar [2].

From the view point of practical applications and the theory of evolutionary algorithms, an important problem is the location of many local extrema and while the location of a global extremum at the same time. It is worth mentioning that the location of a global extremum using the evolutionary algorithm involves an element of probability. Only when we have a priori information, for instance of unimodality of the objective function, is the location of a global extremum certain.

The authors propose an additional procedure called a "linear projection" which consists in the random selection of a direction in the analysed space and the searching for extrema in this direction. In such case, each function of several variables will change into the function of one variable. An additional assumption of the proposed method is the box-type limitation (interval constraints on each variable). This approach enables exploration of the analysed space and location of many extrema at the same time.

2 Description of the method

Let us assume, that the following problem is studied:

$$\max_{x \in X} F(x) \tag{1}$$

where

x	vector $[x_1, x_2, \cdots, x_n]$ of decision variables,
x_i	real or integer number
F(x)	objective function
X	set of admissible solutions

Let us assume that the set of admissible solutions is a box, defined in the following way

$$X = \{x : a_i \le x_i \le b_i ,\, x = [x_1, x_2, \cdots, x_n]\} \tag{2}$$

If in the set X we choose a point $x_0 = [x_{01}, x_{02}, \cdots, x_{0n}]$, $x_0 \in X$ and a direction $w = [w_1, w_2, \cdots, w_n]$, then an arbitrary point in space X can be represented by the relation

$$x_i = x_{0i} + \lambda\, w_i \tag{3}$$

where λ is the coefficient.

Let us assume also that components of the vector of a direction satisfy the condition $w_i \in < -1, +1 >$.

If x_{0i} and w_i are known, then substitution of coordinates given in formula (3) to the objective function transforms the function of many variables into the function of one variable. For given x_{0i} and w_i the range of variations of coefficient λ can be determined by substituting relation (3) into (2) and using the principle of conjunction of inequality. Finally, we obtain

$$\lambda \in < A, B > \tag{4}$$

where

$$A = \max A_i \tag{5}$$

$$B = \min_{i=1 \div n} B_i \tag{6}$$

Values of A_i, B_i are calculated from the following relations

$$\begin{aligned}
&\text{if } w_i > 0 \\
&\quad A_i = (a_i - x_{0i}) / w_i \\
&\quad B_i = (b_i - x_{0i}) / w_i \\
&\text{else} \\
&\quad A_i = (b_i - x_{0i}) / w_i \\
&\quad B_i = (a_i - x_{0i}) / w_i
\end{aligned} \tag{7}$$

Scanning of segment <A, B> cannot be accomplished by the method of golden ratio search or Fibonacci search because function F(x) on a selected cross section may not be unimodal [3].

Bearing in mind that the aim of scanning is to approach an extremum on the cross section and not its precise location, the following procedure is proposed:

1. A number of directions is assumed and a point x_0 is randomly selected from the population.
2. Coordinates for each direction $w^k = [w_1^k, w_2^k, \cdots, w_i^k, \cdots, w_n^k]$ are randomly selected from the interval $w_i \in < -1, +1 >$.
3. For each direction $k = 1 \div p$, the range of variability $\lambda^k \in < A^k, B^k >$ is set up according to relations (5) and (6).
4. Segment $<A^k, B^k>$ is searched with fixed step size of $\Delta\lambda^k$ (constant number of steps – m). The number of steps is assumed by the algorithm designer. This can be, for instance, number m=10, $\Delta\lambda^k=(B^k-A^k)/m$.
5. The location of points x^j and corresponding objective function y^j at the beginning of the interval is memorised if a subsequent value is smaller or when in the sequence of points there is a change from an increasing to decreasing sequence. In this way, all points near a local maximum or in a saddle will be located in the interval $<A^k, B^k>$.
6. Points obtained in this way are added to the population.

The proposed procedure of linear projection enables exploration of the analysed space at each calculation step. It is worth mentioning that it is possible to reach the zone of another extremum, by sampling directions because a subset measure of such area is bigger than zero.

The optimisation procedure starts with the initiation of population, assuming an initial size of the population. In the selection of parents, a generator of pseudo-random numbers with uniform distribution is used taking into account relation (2).

The other procedures, like crossing or mutation, are carried out according to the known methods [1, 4]. In the mutation procedure it may happen that one of the bounds defining the box – relation (2) will be exceeded. In this case, a relevant gene value assumes the value of the bound, which was exceeded.

An additional comment is required on the selection procedure. If a sampled point x_0 is near a maximum then selection of an arbitrary direction will cause that a value of objective function not worse than in point x_0 is found. The population of points close to the extrema will increase as a result of all further calculations. The proposed projection procedure will not bring any progress if we apply hard selection to the total population because the calculations will be convergent to one of the local extremum. Having in mind the above arguments and that the coupling of the genetic algorithms and clustering techniques was a subject of many studies before with positive results, e.g. [6,7], it is proposed to carry out the selection according to the following procedure:

1. Let us assume a number of clusters. Since we do not know a priori the number of local extrema, the procedure can be either a pruning or constructive algorithm.
2. Let us assume a limit of individuals in the cluster. In calculations 10 was taken.
3. Clustering procedure can be realized using one of the known methods of cluster analysis. In this work Kohonen’s method was used [5].
4. If the number of individuals in the cluster is smaller than the limit, the selection of individuals from the analysed cluster does not occur.
5. If the number of individuals in the cluster is bigger than the limit, then hard selection takes place for a given cluster to the size corresponding to the limit.
6. Beside individuals from clusters after selection, we add to the population the position of cluster centres.

3 Example of calculations

To verify the proposed approach, objective functions of two variables with three local extrema significantly distant from each other were taken. The objective function in the area between the extrema is relatively flat. The objective function has the form:

$$F(x) = \sum_{k=1}^{3} h_k \exp\left(-\left(x_1 - c_{1,k}\right)^2 / \sigma_{1,k} - \left(x_2 - c_{2,k}\right)^2 / \sigma_{2,k}\right) \tag{8}$$

Coefficients h_k, c_{1k}, c_{2k}, $\sigma_{1.k}$, $\sigma_{2.k}$ in equation (8) are given in **Table 1** and **Figure 1** shows the function defined by relation (8). The analysed area is <-5, 5> × <-5, 5>.

Table 1. Coefficients in equation (8)

k	h	$c_{1,k}$	$c_{2,k}$	$\sigma_{1.k}$	$\sigma_{2.k}$
1	1.143165	1.336394	-3.220540	0.3024337	1.004549
2	1.177776	0.3898903	0.885901	1.4627276	0.0633130
3	1.8826264	-3.343724	-3.899728	0.1032334	1.044127

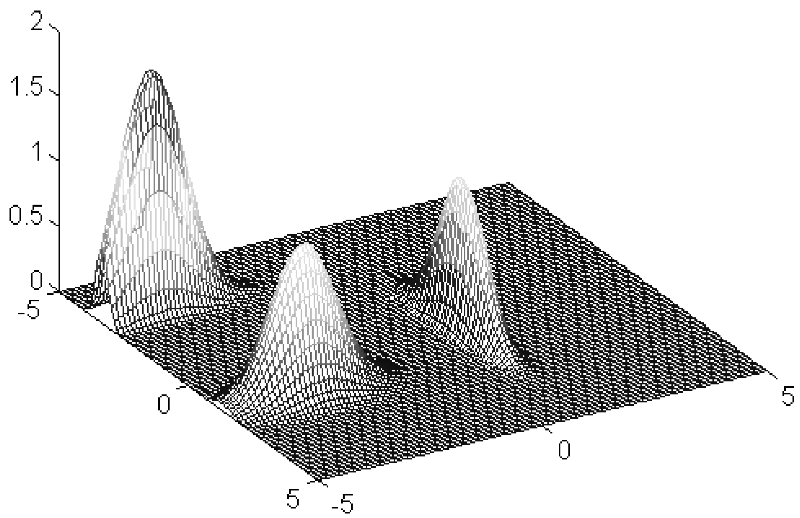

Figure 1. Three-dimensional image of the objective function.

On basis of **Table 1** and the form of function (8), one can easily determine the extrema position and their values. The following was assumed in the calculations:

- number of parents 5,
- coding floating points,

- number of steps in a direction 10,
- number of directions 10,
- the mutation ratio 0.1
- one-point crossing,
- crossing ratio 0.2,
- limit of individuals in a cluster 10,
- number of clusters 4 or 3
- number of generations 20.

Figure 2 shows the location of points after 10 generations. Every calculation, irrespective of the number of clusters, actually located all three extrema.

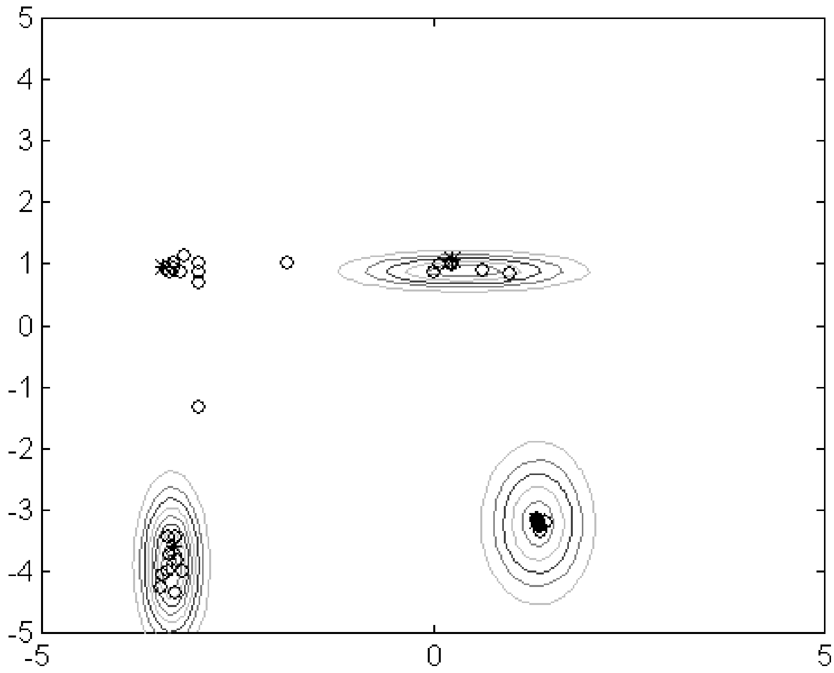

Figure 2. Location of extrema and clusters after 10 generations (four clusters)
o – location of individuals in the clusters, * - location of cluster centers,
solid lines – lines of constant function value.

4. Concluding remarks

1. A new procedure of evolutionary algorithms called "linear projection" whose aim is to better explore the set of admissible solutions and to detect many local and global extrema is proposed in the paper.
2. The proposed procedure should be combined with traditional procedures such as mutation, crossing and selection.
3. It is recommended to apply in the selection procedure, the clustering technique with the limit of individuals in the clusters.
4. The method was presented taken as an example the function of two variables with three separated extrema.

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