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A HYBRID GENETIC APPROACH TO THE HYDRAULIC CONDUCTIVITY IDENTIFICATION IN EARTHEN DAMS

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ABSTRACT: We present an idea of a hybrid optimization algorithm for multidimensional problems with many local minima. We are motivated by the problem of identification of parameters in the Swartzendruber formula. This formula describes the process of prelinear water filtration through cohesive soils. The problem (formulated as an optimal control one) seems to be an example of such multidimensional problem with many local minima. The proposed hybrid algorithm consists of two components: a genetic method and the Törn global optimization method. The components are executed alternately, forming a special kind of a "pulsating" algorithm with diminishing and growing of the area of searches.

The monitoring of the earthen dam stability has the great engineering importance. Determination and updating of hydraulic parameters of the central protection screen (see Fig.1) which decides of the dam tightness, play the crucial role in this process. Protection screens are usually made of cohesive soils, such as clays and silts.

The mathematical model of the filtration in the full saturated cohesive soil may be completed as follows:

$$\begin{cases} \alpha(t)e(x)\dfrac{\partial h}{\partial t} - div\Big(v\big(p, Dh_p\big)\Big) = g(t,x) \text{ in } Q \\ h(0) = h_0 \text{ in } \Omega \\ h|\Sigma_1 = h_b \text{ on } \Sigma_1, \quad v\big(p, Dh_p\big)\boldsymbol{n} = q \text{ on } \Sigma_2 \end{cases} \tag{1}$$

The filtration velocity v is described by the modified Swartzendruber formula [4]:

$$v^i\big(p(t,x), Dh_p\big) = \phi\big(\tilde{p}(t,x), |Dh_p|_L\big)\sum_{j=1}^{3} l_{ij}(t,x)\frac{\partial h}{\partial x_j}$$

$$|\eta|_L = \sum_{i,j=1}^{3} l_{ij}\eta_i\eta_j$$

$$\phi(\tilde{p}, s) = \begin{cases} M\left(1 - \dfrac{s_0}{s}\left(1 - \exp\left(-\dfrac{\Theta E}{s_0}\right)\right)\right), & \text{if } E < s \\ \left(\dfrac{M}{E^2}\left(s_0 - (s_0 + \Theta E)\exp\left(-\dfrac{\Theta E}{s_0}\right)\right)\right)s & \\ +M\left(1 - \dfrac{2s_0}{E} + \left(\dfrac{2s_0}{E} + \Theta\right)\exp\left(-\dfrac{\Theta E}{s_0}\right)\right), & \text{if } 0 \le s \le E \end{cases} \tag{2}$$

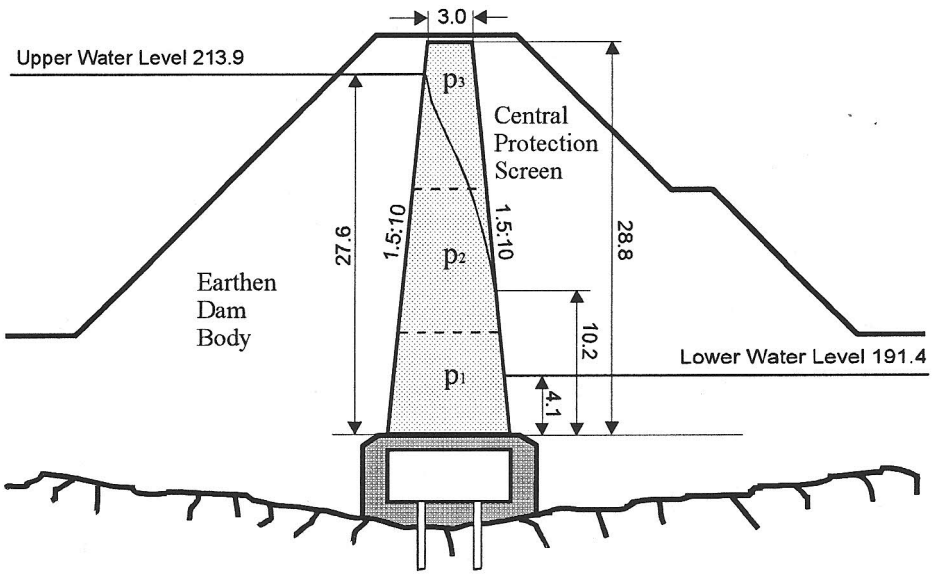

Fig. 1The typical cross section through the earthen dam

where h_p is the piesometric pressure, v denotes the filtration velocity in the region $\Omega \subset \Re^3$, $Q = \Omega \times [0,T]$, $\Sigma_1 = \partial\Omega_1 \times [0,T]$, $\Sigma_2 = \partial\Omega_2 \times [0,T]$, $\partial\Omega_1 \cup \partial\Omega_2 = \partial\Omega$, $\int_{\partial\Omega_1 \cap \partial\Omega_2} d\sigma = 0$, $\alpha(t)e(x)$ is the elastic soil capacity, $\boldsymbol{n}$ is the unit outlet normal to $\partial\Omega_2$, q, h_b and h_0 represent respectively, the boundary inflow, the fixed boundary pressure and the initial pressure in Ω. Moreover,

$$\begin{aligned} p(t,x) &= \left(\{l_{ij}(t,x)\}, \tilde{p}(t,x)\right) \\ \tilde{p}(t,x) &= \left(M(t,x),\ s_0(t,x),\ \Theta(t,x)\right) \end{aligned} \quad \text{for } (t,x) \in Q \tag{3}$$

are hydraulic parameter functions which defines the local permeability of the soil, according to the formula (2) (see [4,5] for details).

In the case of earthen dam protection screens, the initial value of $p(0,\cdot)$ may be evaluated by laboratory tests which can be made only during the construction. Identification or updating of $p(t,\cdot), t > 0$ may be done by the proper inverse problem solution based on permanent measurements of the piesometric height distribution h_m in the wall's interior.

The above inverse problem can be formulated as the optimal control one (cf. [1]).

$$\inf_{p \in \mathcal{P}} \left\{ \mathcal{F}\left(C(h_p)\right) + G(p) \right\} \tag{4}$$

where h_p satisfies the model (1)-(3). Here C is the observation operator and $\mathcal{P}$ denotes the compact set of a metric space of admissible parameter functions.

Both, state equations (1)-(3) and functional (4) are replaced by their discrete versions using mixed Finite Element/Finite Difference technology (see [1,2,5,7]) in order to provide the effective computations. Mathematical results concerning existence of the above problems (1)-(3), (4) and their discrete versions have been proved in [2,4,5] assuming the regular class of functionals $\mathcal{F}$ and G. The particular form of (4) which have been used by numerical tests is the following:

$$F\left(C\left(h_p\right)\right)+G(p)=\sum_{i=1}^{N}\sum_{j=1}^{K}\left(\left|h_p\left(t_j,x_i\right)-h_m\left(t_j,x_i\right)\right|^2 w\left(t_j,x_i\right)\right)+|p-p_d| \tag{5}$$

The first part of the right hand side of (5) represents the weighted distance between the observed (h_m) and simulated (h_p) piesometric pressures in piesometer locations $x_i \in \Omega, i=1,...,N$ and in time instances $t_j \in [0,T]$, $j=1,...,K$. The second is the penalty which grows if the parameter vector is far from the desired one p_d. The weighting function w is set rather heuristic and such that the first expression strongly dominates over the second one. The second expression plays also the role of the regularisation factor.

The mathematical results in [1,2] and the current numerical experience (see [1,7]) lead to the conclusion that the inverse problem under consideration has multiple solutions, then it is ill posed one. The next remark which comes rather from a heuristic consideration is that the solutions lie in several distant subregions, and the solution points are placed rather dense in each subregion.

The exemplary results which exhibit that the considered inverse problem is ill posed because of many local minima are presented below. The results concern the case of one dimensional flow under a testing embankment (the data were obtained from the Warsaw Agricultural University) with $N=3$ and $K=1$. The dimension of $\mathcal{P}$ is 9. The vector p consists of 9 components M_i, θ_i, s_{0i}, i=1,...,3. We denote $M_i' = M_i \cdot 10^7$ [m/s].

	Example 1			Example 2			Example 3		
	M_i'	θ_i	s_{0i}	M_i'	θ_i	s_{0i}	M_i'	θ_i	s_{0i}
i=1	0.362	0.830	2.700	0.371	0.800	3.214	0.298	0.708	2.269
i=2	0.462	0.872	2.842	0.394	0.801	2.565	0.441	0.851	2.778
i=3	6.189	0.800	2.502	6.253	0.801	2.482	6.230	0.801	2.501
Final value of $\mathcal{J}$	$\mathcal{J}=7.2\cdot10^{-5}$			$\mathcal{J}=2.0\cdot10^{-4}$			$\mathcal{J}=1.1\cdot10^{-4}$		

The values obtained by laboratory tests (during construction of the embankment) are: M' =(0.38, 0.45, 6.20), θ =(0.86, 0.86, 0.80), s_0=(2.8, 2.8, 2.5).

Table 1 Exemplary results of the hydraulic parameters identification

The standard variable metric method Migrad (from MINUIT - CERN library) was applied. Three local minima were found. The situation becomes worse if $K > 1$, $N > 3$ or we consider the two or three dimensional flow.

The numerical tests exhibit that the applied variable metric method stops in many points of the solution domain (with or without convergence of Migrad). Some of these points can correspond to local minima, some may follow from bad conditioning. The obtained results strongly depend on starting points (see [1,7]).

Although we can improve the conditions of the functional (4) (by e.g. enlarging the penalty G) in order to obtain the unique solution, there is more interesting to find as much regions of solutions as possible, and then the local minimizers therein. The process of finding multiple solutions in large multidimensional set $\mathcal{P}$ in reasonable time needs a special kind of optimization algorithms. Some proposals of such algorithms were given in [8] and they concerned stabilisation (less dependence of starting points) via hierarchic optimization and also parallelisation and distribution of computations in a computer network. Here we propose to involve genetic methods to the optimization algorithm.

The concept of the hybrid algorithm for finding local minima

One of methods that can be used to deal with many local minima is to review large areas of the domain, by running many local optimization processes in various parts of it. The search can be fastened by a proper coarse grain parallelisation: a basic variant of such an algorithm (called MSP - see [8]) consists in executing parallely many scalar nonlinear programming processes starting from various points. This method in some way can also deal with the bad conditioning. However, the MSP has the following disadvantage: when the dimension of the optimization task is big (say tens or hundreds or more), the domain of searches becomes too large even for parallel and distributed computations. Referring to the considered identification problem, if the dimension of $\mathcal{P}$ is m and we would like to have l values for each component of the parameters vector p, then the total number of the initial starting points is equal to l^m. Searching through millions of points using the MSP becomes ineffective even if we distribute computations in a small (tens of workstations) network. Some preliminary steps to recognize those parts of the domain, where existing of minima is more probable would be desired.

We propose to apply a hybrid algorithm in order to determine these areas. The two most important components of it are an evolution algorithm and the Törn global optimization method [9,10].

The idea of the Törn method is based on the observation, that when we start local optimization processes (standard methods of the convex analysis) from many points (called after Törn the *global points*), we often obtain the same local minimum. Instead of wasting time for exact computations from each starting point, we can compute only several iterations of a very simple optimization method. Obtained points are called the *intermediate points*. A set of intermediate points that belong to one local minimum is called a *cluster*. For further exact computations, we can choose one or two points from each cluster.

However, it may happen, that a cluster in fact contains more than one local minimum. Thus, the process of finding intermediate points from the global ones can be recurrent. When we choose new global points in the recognized cluster and proceed the Törn method once again, then we can obtain new clusters.

The stop criterion should be complex:

- No new clusters in the consecutive Törn step were found: then exact computations (standard methods of convex analysis together with proper standard stop criteria) from one or two points in each cluster should be executed.
- The distances between clusters are smaller than a certain threshold.
- The maximum time of computations (or the maximum number of function evaluations) was exceeded.

An important question in the Törn method is how to determine clusters. One of possible ways is to consider the density ρ of intermediate points. In order to recognize a cluster we choose a seed point x_0 and then scan $\mathcal{P}$ in a hyperspherical neighbourhood $S(r)$ of x_0 with a radius r. The radius is increased while $\rho > \tilde{\rho}$, where $\tilde{\rho}$ stands for a threshold and ρ can be computed as:

$$\frac{\text{number of intermediate points}}{V(r)},$$

$V(r)$ is the capacity of $S(r)$. This method is described in [10]. Another method of clusters determination is to divide $\mathcal{P}$ into hypercubes (that will create a grid) and to count number

of intermediate points in each hypercube. The grid can be condensed in proper parts of $\mathcal{P}$. The more accurate description of this problem is beyond our considerations in this paper.

The disadvantage of the Törn method is that it can detect only these $m+1$ (m stands for the number of parameters to be found) dimensional "valleys", in which we have chosen global points. The method has not the property of crossing local "crests". Therefore we propose to apply the Törn method together with some steps of an evolution algorithm. Evolution algorithms [3] in general are able to cross such "crests". Usually they give a set of points (population) that can form clusters like in the Törn method. Clusters can be recognized by the same techniques as previously described. We propose to use the two mentioned methods alternately: the first stage consists in a genetic algorithm, the second stage in the Törn method, the third stage consists again in a genetic algorithm.

The flow diagram of the hybrid optimization algorithm is presented on Figure 2.

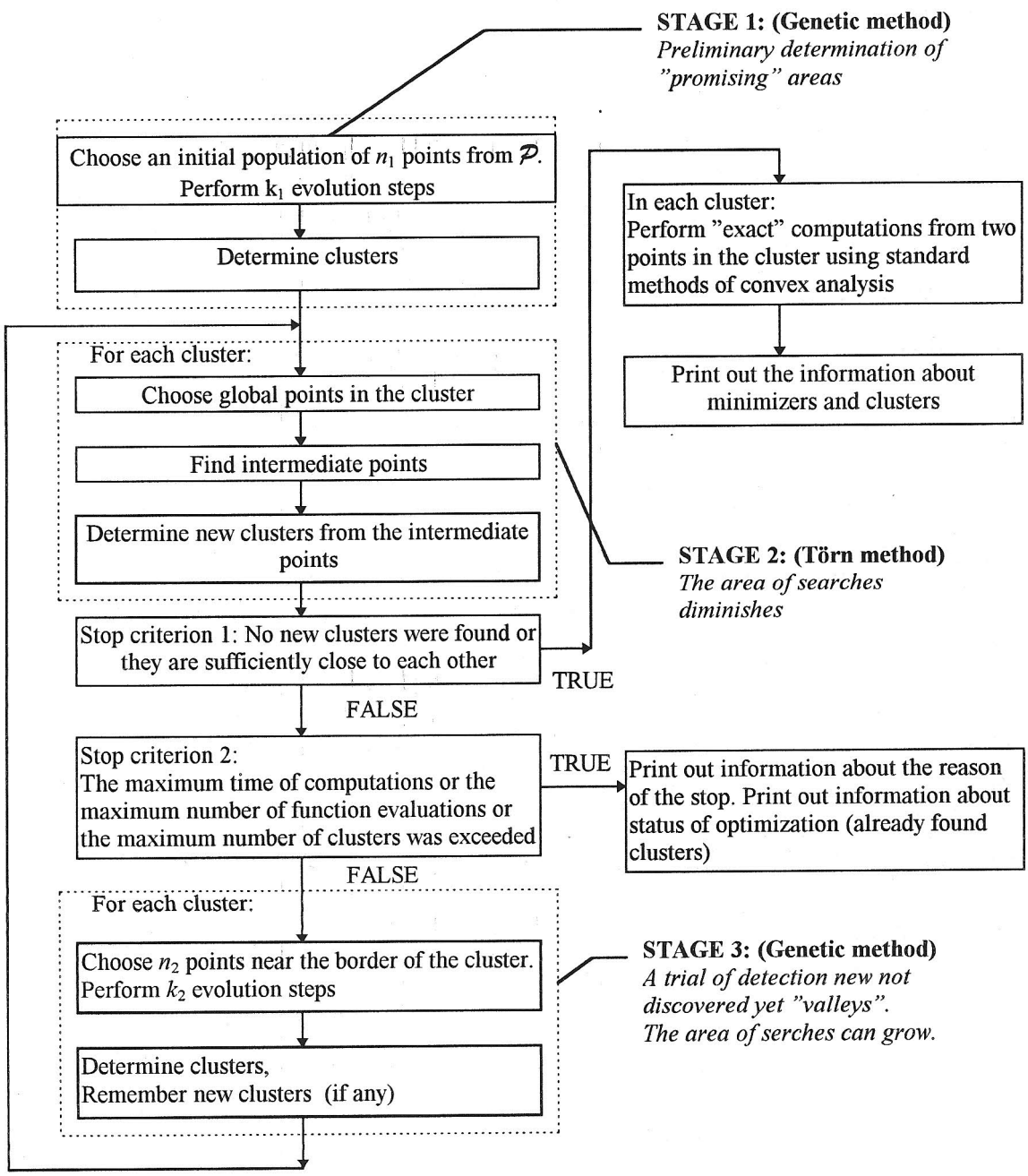

Fig. 2 The block diagram of the proposed hybrid algorithm

We expect, that the first stage of the proposed method will result in a rough determination of these parts of the domain, where there can exist one or more local minima ($m+1$ dimensional "valleys"). The property of implicit strong parallelism of a genetic algorithm gives a chance that the multidimensional domain will be searched relatively quickly.

In the next stage of the algorithm - the Törn method - we would like to determine more precisely internal "valleys" in the previously recognized areas. The possible greater precision follows from the fact that the more accurate (than genetic ones) methods of conventional nonlinear programming are used. The searched area is diminished in this stage.

However, the second stage does not guarantee that all "internal valleys" will be found. When we want to find as much valleys as possible, we can use genetic method once again. This is the third stage of the proposed algorithm. This time the initial population is generated near the borders of the recognized valleys. After some evolution iterations we can check if the new generations concentrate in "old" valleys, or they tend to another, not discovered yet ones. This stage can result in the growth of the searched area.

The idea of the proposed algorithm is illustrated also in Figure 3. The situation after the stage 1 is depicted in Figure 3A. Figure 3B presents stages 2 and 3.

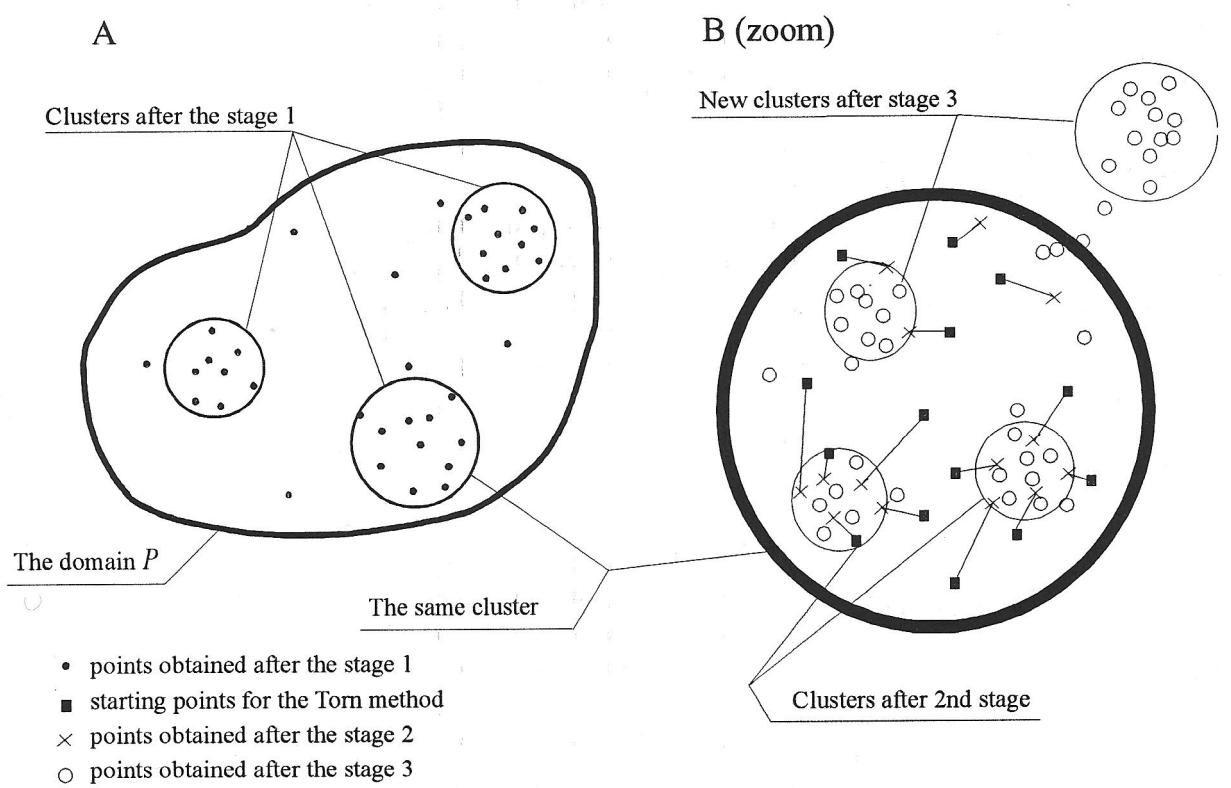

Fig. 3 Illustration of the three stages of the proposed algorithm

The second and third stages can be executed alternately as long as new "sub-valleys" are discovered.

The genetic process to be employed as a tool for optimization consists in breeding a population of individuals which represent a set of points in the domain $\mathcal{P}$. The starting population evolves by standard genetic mechanisms as reproduction with crossover, mutation and selection.

The gene code Q of a single individual can be considered as the catenation $q_1 q_2 \ldots q_m$. Parts q_i, $i=1,\ldots,m$ result from an integer coding of a single coordinate of a point in $\mathcal{P}$. Such coding is obtained after covering the domain $\mathcal{P}$ by an arbitrary dense m-dimensional regular mesh. The i-th coordinate of a point $p \in \mathcal{P}$ is represented by q_i, which is the integer number of this grid cell that contains p, counted in the i-th versor direction. The binary representation of q_i is used in genetic operations on the individual p. The approach presented above is the classic one (see [3]), however we do not exclude future refinements.

Final remarks

We expect two main advantages of the application of the genetic method to the considered identification problem.

The first one is relatively quick preliminary determination - due to strong implicit and possibly explicit parallelism (in the sense of Goldberg [3]) of evolution algorithms - of these parts of $\mathcal{P}$ in which local minima can exist.

The second advantage is connected with the property of crossing local "crests" that gives a chance to obtain more local minima. Moreover, approaching to minima from many sides (many points in a cluster) together with the statistic character of genetic methods gives also a chance to overcome (at least partially) bad conditioning of the optimization task.

We suggest that the proposed method can be also applied for other problems with many local minima. An important class of such problems - which have the great significance - are molecular configuration problems. They have hundreds or thousands parameters and typically many local minima with function values very close to the global one. Moreover local minima often have small basins of attraction [6]. It seems, that the described "pulsating" hybrid method can be helpful in these cases.

This paper contains only the idea of the hybrid algorithm, which is not complete and tested yet. There are many questions, that are still open. The proper investigations will be carried on. Also the mathematical analysis of properties of the proposed algorithm is out of our consideration here. We suggest, that such analysis should be provided basing on statistic methods.

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